Summary of

Radiative processes relevant to radioastronomy

Disclaimer:

These are SLIDES and not lecture notes

The topics in these slides have never been discussed @ exams.

However, a solid knowledge of the (astrophysics) is necessary to achieve a proper and fruitful understanding of the various aspects/phenomena/bodies in the science plan of this course

Continuum processes: accurate description (& proper maths as well) can be found in

- Rybicky & Lightman "Radiative processes in Astrophysics" Chaps 3-4-5
- Longair "High Energy Astrophysics" Part II, Chaps 5,6,7,8,9

Disclaimer:
These are SLIDES and not lecture notes

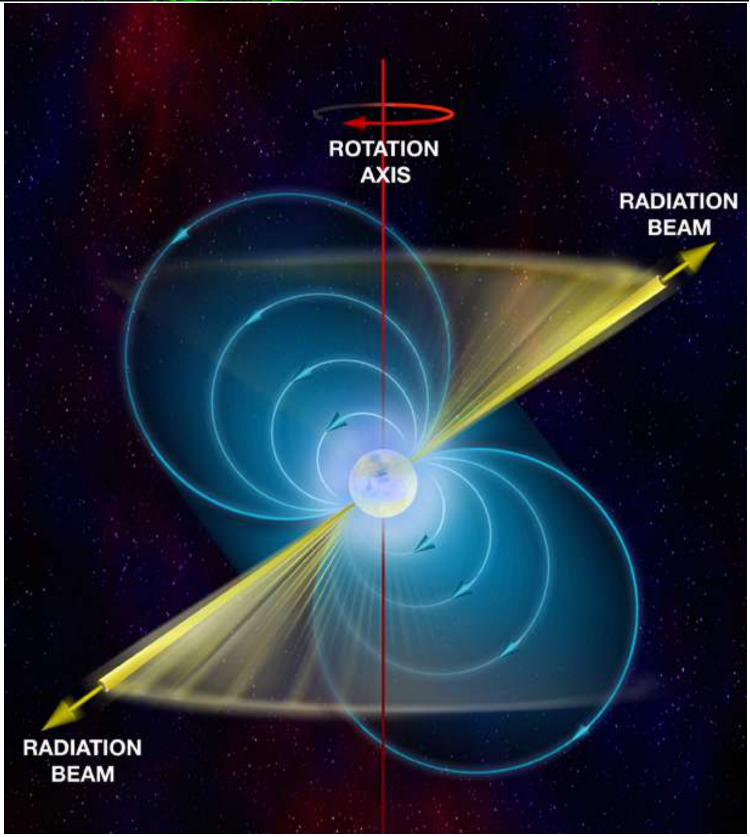
Synchrotron:

Radio Galaxies (RLQSOs, BL Lacs, Blazars, ...)

Microquasars

SNR

Pulsars (PWN)



Thermal Bremsstrahlung:

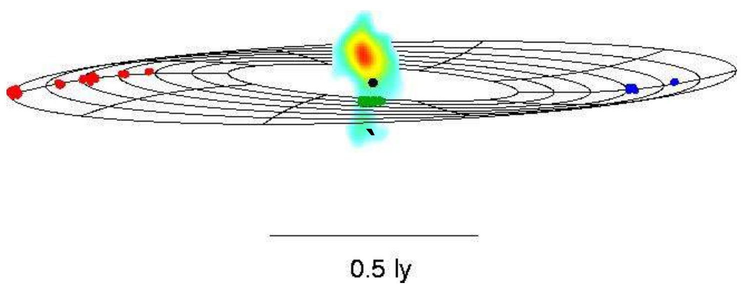
HII regions (also in galaxy clusters: T & NT)

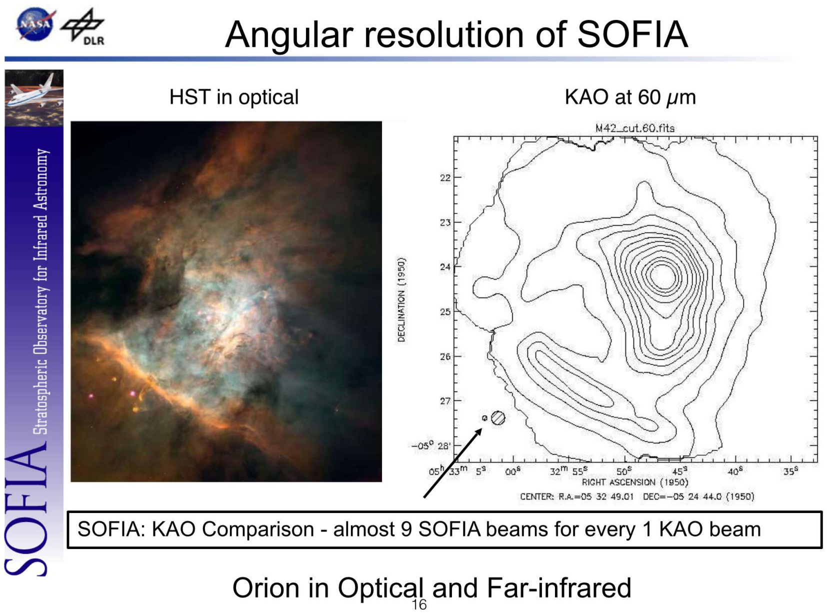
Line emission:

HI in spiral galaxies (CO)

Masers (in SFR or in evolved stars)

Megamasers (in external galaxies)





Radiation from moving charges

Differentiation of the Liénard-Wiechert potentials (easy but lengthy) produces the radiation fields at a position r and a time t (computed at the "retarded time" t_{ret} and corresponding position r_{ret})

Let $\vec{\beta} \stackrel{def}{=} \frac{\vec{u}}{c}$ and then $\kappa \stackrel{def}{=} 1 - \vec{n}\cdot\vec{\beta}$

$$\vec{E}(\vec{r},t) = q\left(\frac{(\vec{n}-\vec{\beta})(1-\beta^2)}{\kappa^3 R^2}\right) + \frac{q}{c}\left(\frac{\vec{n}}{\kappa^3 R}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]\right) \qquad \vec{B}(\vec{r},t) = \vec{n}\times\vec{E}(\vec{r},t)$$

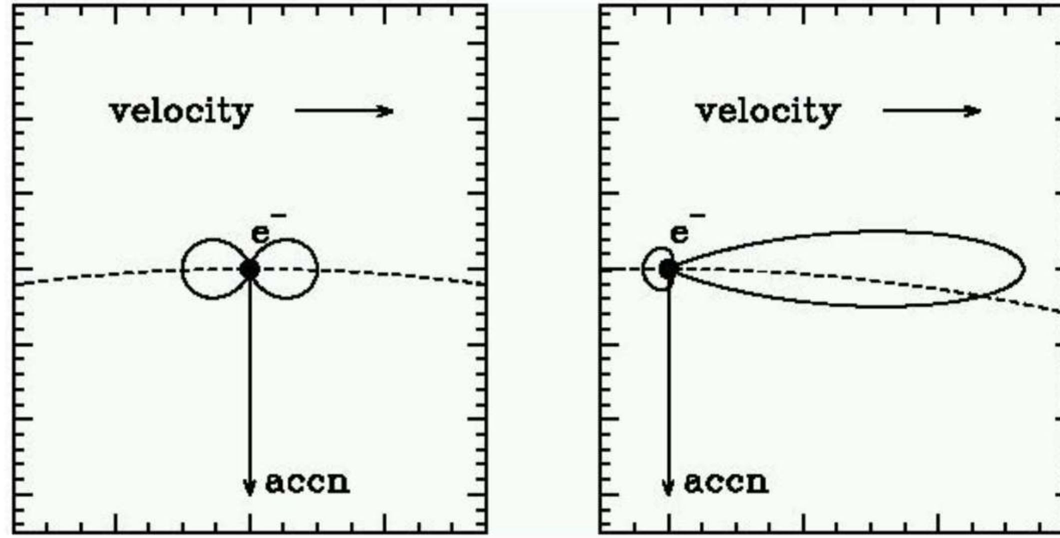
1. Coulomb's law holds for $\beta \ll 1$ & no acceleration; $\vec{E}$ field points to current position of the charge
2. In case of acceleration the radiation field [at large distances from the charge] is then

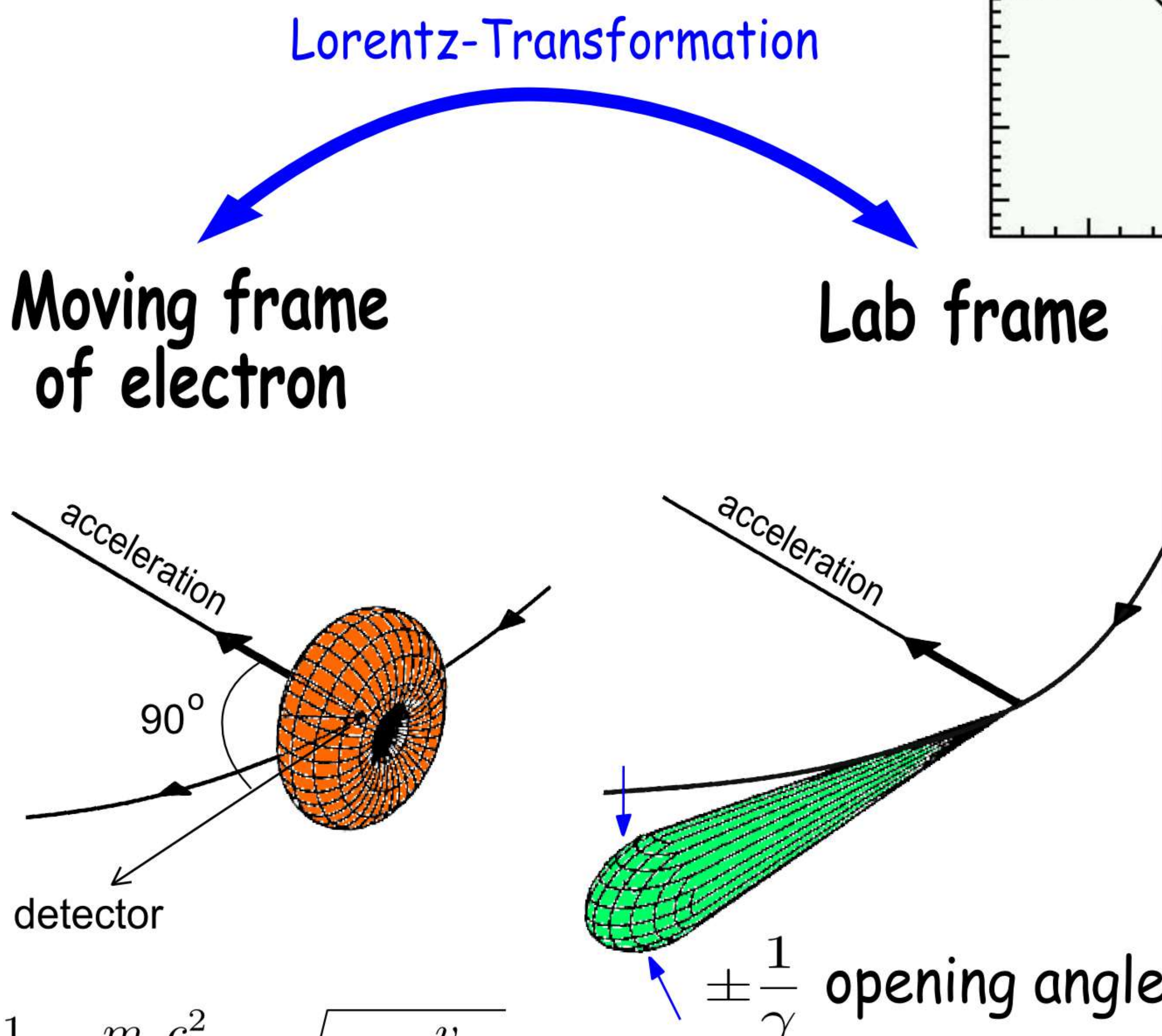
$$\vec{E}_{rad}(\vec{r},t) = \frac{q}{c}\left(\frac{\vec{n}}{\kappa^3 R}\times[(\vec{n}-\vec{\beta})\times\dot{\vec{\beta}}]\right) \qquad \vec{B}_{rad}(\vec{r},t) = \vec{n}\times\vec{E}_{rad}(\vec{r},t)$$

A consequence of the retarded potentials (Lienard-Wiechert)
Any **accelerated charge** emits radiation following

$$-\frac{dE}{dt} = \frac{2q^2a^2}{3c^3}$$

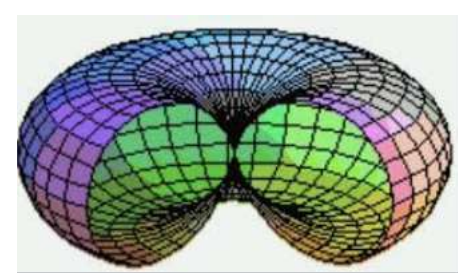
Longair: High Energy Astrophysics, Chap 8
(evolution of the spectrum, energetics are missing)





For a relativistic particle, the Doppler effect enhances the energy of the emitted photon in the direction of v. The radiation within a 1/γ cone wrt the istantaneous direction of the particle is largely amplified (relativistic beaming)

$$\frac{1}{\gamma} = \frac{m_o c^2}{E} = \sqrt{1-(\frac{v}{c})^2}$$



Emission pattern of an oscillating electron (ā is vertical)

Relativistic charges (electrons & positrons) in a magnetic field

In a uniform field $$-\frac{d\vec{p}}{dt} = \frac{q}{c}\vec{v}\, x\, \vec{H}$$ (Lorentz, n-rel)

Radiated energy

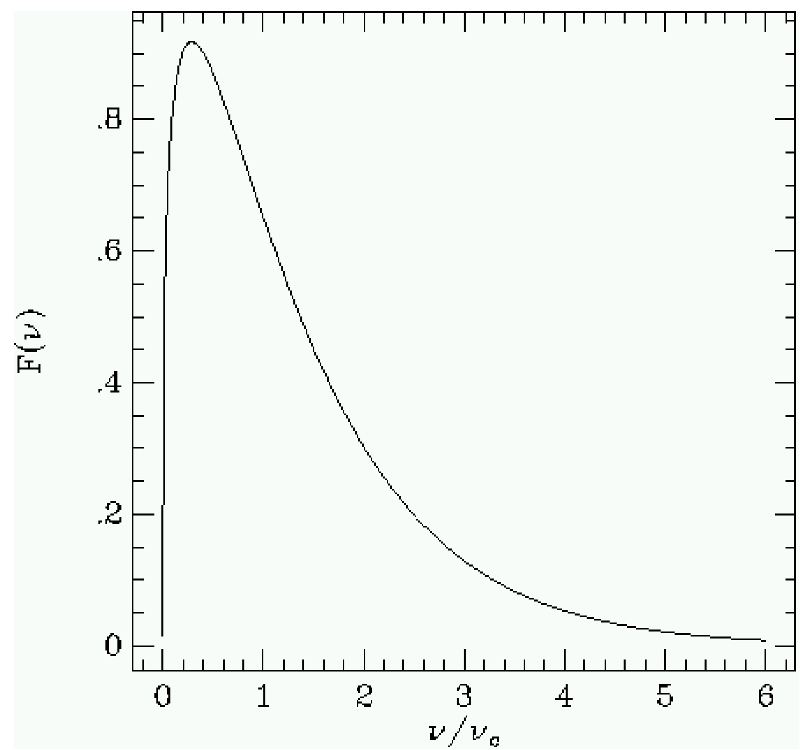
(Larmor, relativistic)

$$-\frac{dE}{dt} = \frac{2}{3}\frac{q^2}{c^3}\left(-\frac{d\vec{p}}{dt}\right)^2\gamma^2 = \frac{2}{3}\frac{q^2}{m^2c^3}\left(\frac{q}{c}\vec{v}\, x\, \vec{H}\right)^2\gamma^2$$

Emission

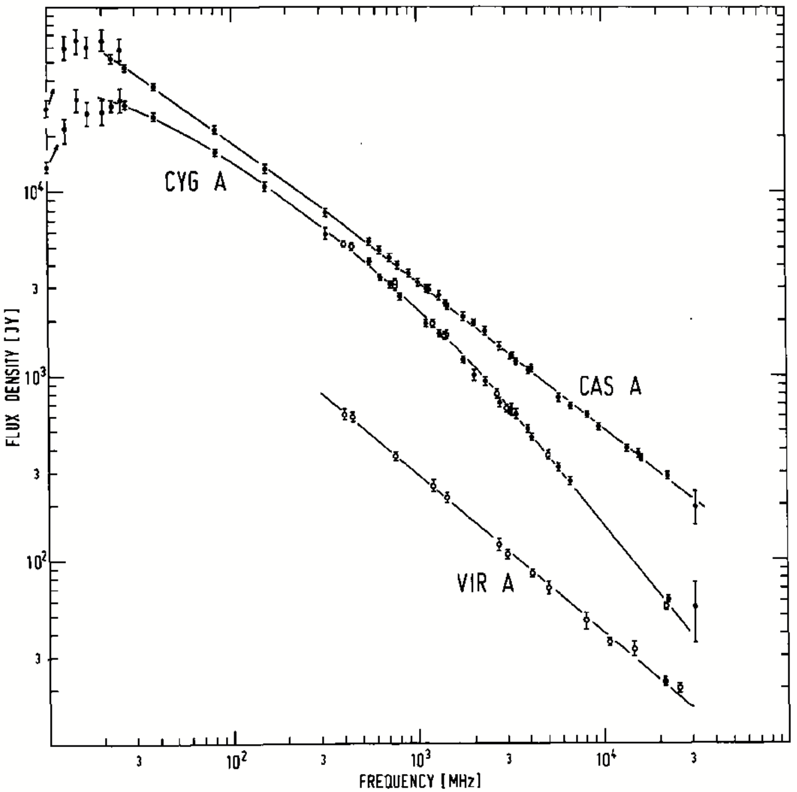
- in a narrow cone — *aperture* $\frac{2}{\gamma}$
- short pulse duration $\tau \simeq \frac{1}{\gamma^2\omega_L} \simeq \frac{5\times10^{-8}}{\gamma^2 H(G)}$
- at a characteristic frequency $\nu_s \simeq \frac{3}{4\pi}\gamma^2\frac{eH}{m_e c}$

$$\nu_s \simeq 4.2\times10^{-9}\gamma^2\left(\frac{H}{[\mu G]}\right) \quad GHz$$



Magnetized plasma: (if e+p, the latter preserve their energy content)

Specific emissivity
$$J_s(\nu) = -\frac{dE}{dt} N(E) \frac{dE}{d\nu}$$



For a given particle energy distribution **(non-thermal)**

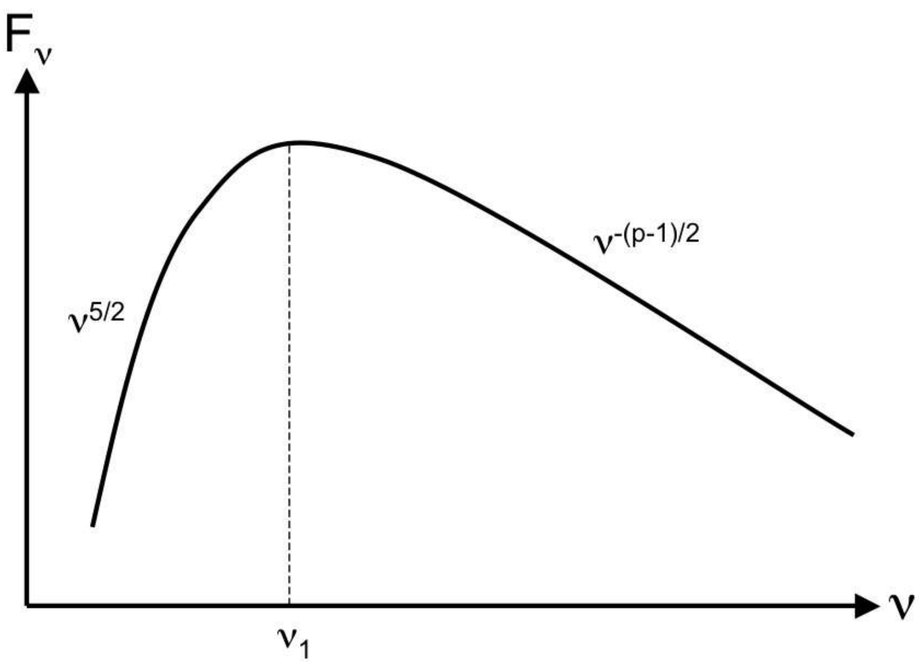
$$N(E)dE = N_o E^{-\delta} dE$$

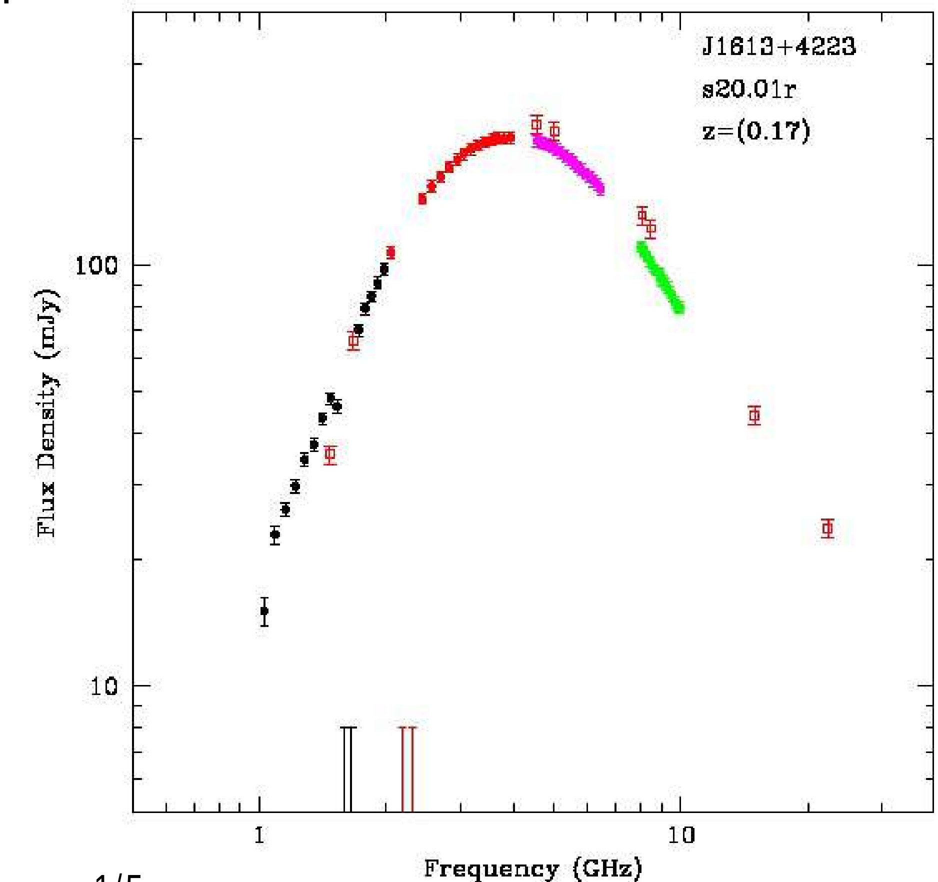
Well known **power-law** emission

$$J_s(\nu) = C_{sync} N_o H^{(\delta+1)/2} \nu^{-(\delta-1)/2} \sim N_o H^{(\delta+1)/2} \nu^{-\alpha}$$

$$\alpha = \frac{\delta - 1}{2}$$ spectral index

Synchrotron self-absorption relevant in **small** plasma bubbles



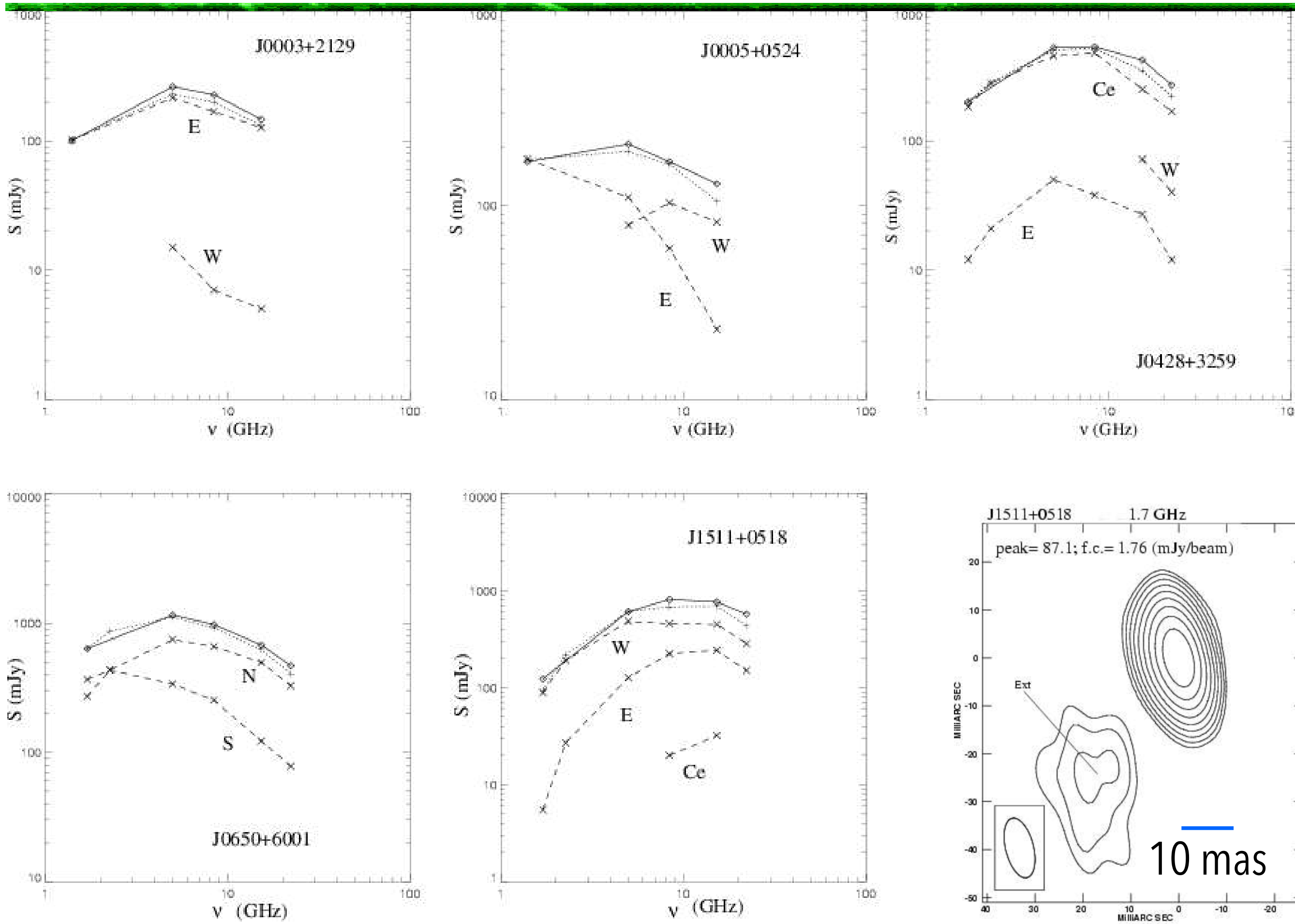


$$\nu_{max} \simeq 2\left(\frac{S_{max}}{Jy}\right)^{2/5}\left(\frac{\theta}{mas}\right)^{-4/5}\left(\frac{H}{mG}\right)^{1/5}(1+z)^{1/5} \quad \text{GHz}$$

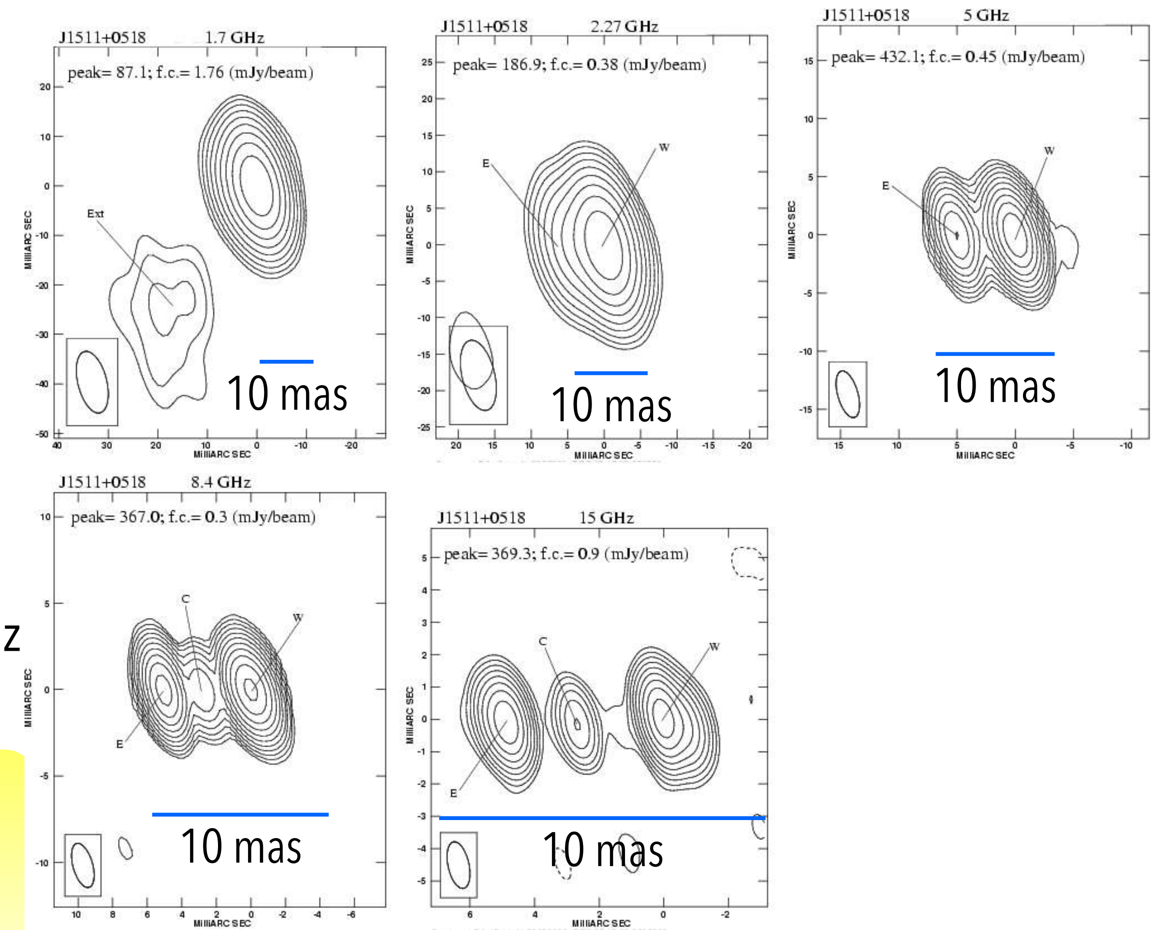
$$H \simeq 0.03\left(\frac{\nu_{max}}{GHz}\right)^{5}\left(\frac{S_{max}}{Jy}\right)^{-2}\left(\frac{\theta}{mas}\right)^{4}(1+z)^{-1} \quad \text{mG}$$

$\nu_{max}, S_{max}, \theta$ measured from observations $\rightarrow$ errors can be large on θ ...assumptions

Summary of Synchrotron emission mechanism



Orienti & Dallacasa 2008, A&A, 487, 855



$$\nu_{max} \simeq 2\left(\frac{S_{max}}{Jy}\right)^{2/5}\left(\frac{\theta}{mas}\right)^{-4/5}\left(\frac{H}{mG}\right)^{1/5}(1+z)^{1/5} \quad \mathrm{GHz}$$

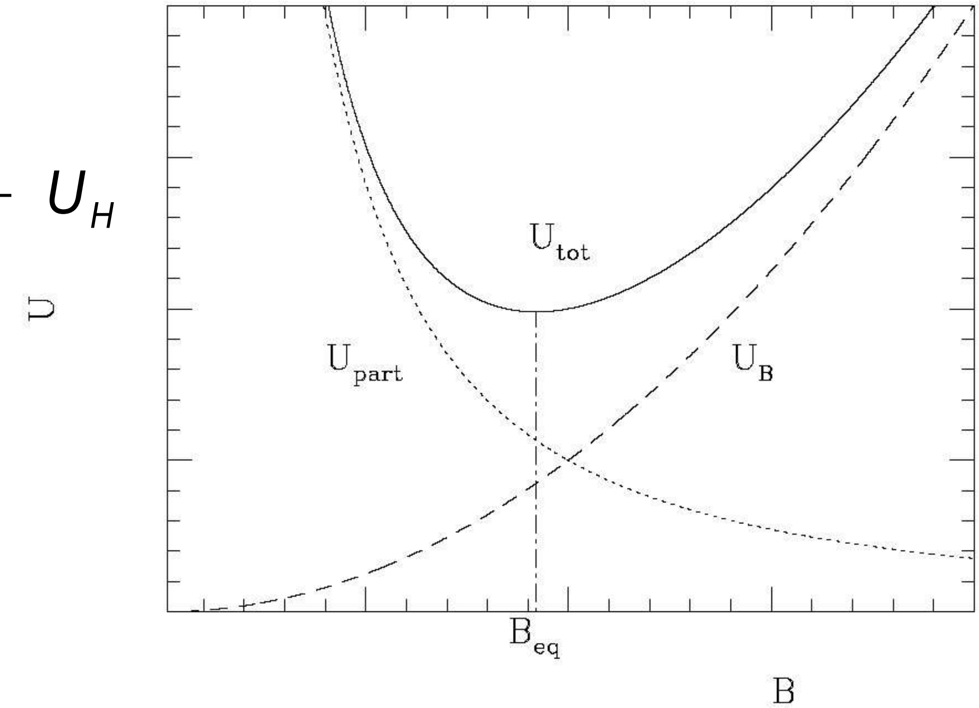
$$H \simeq 0.03\left(\frac{\nu_{max}}{GHz}\right)^{5}\left(\frac{S_{max}}{Jy}\right)^{-2}\left(\frac{\theta}{mas}\right)^{4}(1+z)^{-1} \quad \mathrm{mG}$$

Energetics of a radio source

$$U_{tot} = U_{el} + U_p + U_H = (1+k)U_{el} + U_H$$

$$U_{el} = C_{el} H^{-3/2} L$$

$$U_H = \int \frac{H^2}{8\pi} dV = C_H H^2 V$$



where L and V are Luminisity and Volume; then

$$U_{tot} = (1+k) C_{el} H^{-3/2} L + C_H H^2 V$$

$$(1+k)U_{el} = \frac{4}{3} U_H$$ provides the minimum total energy

$$U_{tot,min} = 2\times 10^{41} (1+k)^{4/7} \left(\frac{L_{1.4GHz}}{Watt}\right)^{4/7} \left(\frac{V}{kpc^3}\right)^{3/7} \text{[erg]}$$

Minimum energy density

$$u_{min} = \frac{U_{tot,min}}{V}$$

which is related to the magnetic field intensity, which is then known as equipartition magnetic field

$$H_{eq} = \sqrt{\frac{24\pi}{7} u_{min}} \quad \text{[Gauss]}$$

Which can be compared with the field found in self-absorbed radio Sources.

Orienti & Dallacasa (2008)

Source	Comp	H (mG)	H_{eq} (mG)	u_{min} erg/cm^{-3} (10^{-4})	p_{min} dyne/cm^{-2} (10^{-4})
J0003+2129	E	33	30	5.0	3.1
J0005+0524	E	-	18	0.75	0.46
J0428+3259	E	1000	34	0.75	0.46
	Ce	59	65	3.9	2.4
J0650+6001	N	29	77	6.0	4.0
	S	10	54	1.5	1.0
J1511+0518	W	104	95	8.3	5.2
	E	1000	70	3.8	2.4
J1459+3337		160	160	24	15

Approximated IC scattering

Longair: High Energy Astrophysics, Chap 9 (9.3)

Let ν, ν' be the energies in the lab or electron reference frames

In the electron RF, photons coming from a small angle to the velocity of the electron are amplified

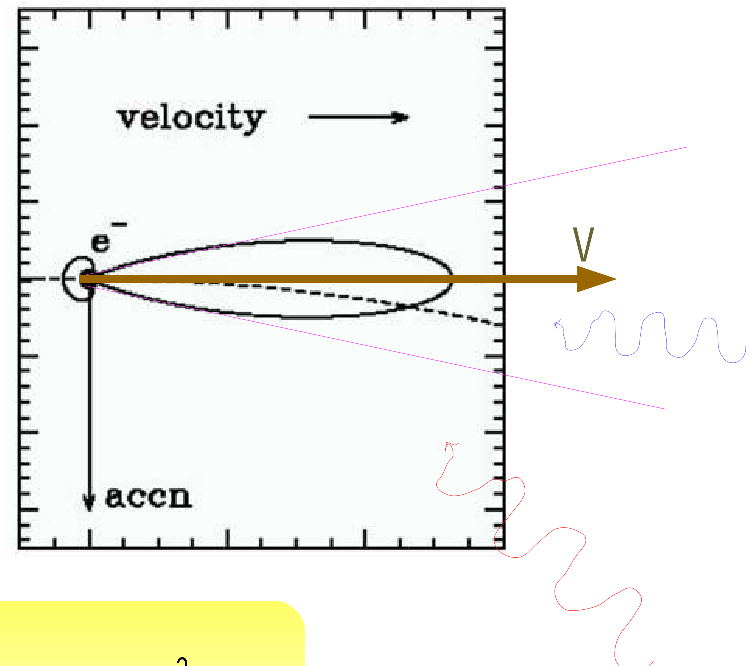
$$h\nu_i' \approx \gamma h \nu_i$$

In the electron RF, if $\gamma h \nu \ll m_e c^2$ Scattering Thomson takes place

$$h\nu_f' = h\nu_i' \approx \gamma h \nu_i$$

In the scattered photon comes out the $1/\gamma$ cone, then it is amplified with an additional γ factor

$$h\nu_f \approx \gamma h \nu_f' = \gamma h \nu_i' \approx \gamma^2 h \nu_i$$



For a plasma in a radiation field, the IC luminosity is

$$-\frac{dE}{dt} \approx E^2 U_{rad} \simeq \gamma^2 U_{rad}$$

U_{rad} can be, at minimum, $0.25\,\mathrm{eV\,cm^{-3}}$ (the CMB!)

N.B. Only a fraction of the total number of interaction increases the energy of the diffused photon!

$\rightarrow$ Geometry is fundamental!

Interaction (scattering) between a relativistic electron and a (low energy) photon

The energy of the photon is increased by a factor $\approx \gamma^2$ at expenses of the kinetic energy of the electron (whose loss is $\approx \varepsilon^2$)

➔Low-energy (radio,mm, sub-mm) photons are shifted to the X-rays (and beyond!)

A population of relativistic electrons provide a modification of the incoming radiation spectrum loosing energy at the rate of

$$-\left(\frac{d\varepsilon}{dt}\right)_{i.c.} = C_{i.c.}\,\varepsilon^2 U_{rad} \quad \text{similar to} \quad -\left(\frac{d\varepsilon}{dt}\right)_{syn} = C_{syn}\,\varepsilon^2 U_H$$

If U_{rad} is known, then U_H can be determined

→ Further **method to measure H: compare L_X and L_R for radio loud objects**

Radio spectrum and its *evolution*: Continuity equation

$$\frac{\partial \rho}{\partial t} + \nabla \rho v = 0 \quad \text{aka} \quad \frac{\partial \rho}{\partial t} = -\nabla \vec{J}_q \quad \text{becomes}$$

$$\underbrace{\frac{\partial N(\varepsilon,t)}{\partial t}}_{\text{particle flow}} + \frac{\partial}{\partial \varepsilon}\underbrace{\left(\frac{d\varepsilon}{dt} N(\varepsilon,t)\right)}_{\text{energy losses}} + \underbrace{\frac{N(\varepsilon,t)}{T_{conf}}}_{\text{leakage}} = Q\underbrace{(\varepsilon,t)}_{\text{injection}}$$

Solutions are complex, dependent on the history of the plasma bubble (particle generation, interaction with the medium, ...)

Particular cases are considered

$\frac{\partial N}{\partial t} = 0$	(quasi) stationary solution, particles do not leave the bubble
$N(\varepsilon,0) = N_o \ \varepsilon^{-\delta}$	initial injection of particles
$Q(\varepsilon,t) = A \ \varepsilon^{-\delta}$	continuous injection of particles (acceleration)
$T_{conf} \simeq \infty$	no particle leakage

Energy losses stand for: Ionization ($\sim c_1$)

Relativistic Bremsstrahlung & Adiabatic Expansion ($\sim c_2\,\varepsilon$)

Synchrotron & Inverse Compton ($\sim c_3\,\varepsilon^2$)

$$N(\varepsilon) = \frac{A\ \varepsilon^{-(\delta-1)}}{(\delta-1)(c_1 + c_2\varepsilon + c_3\varepsilon^2)}$$

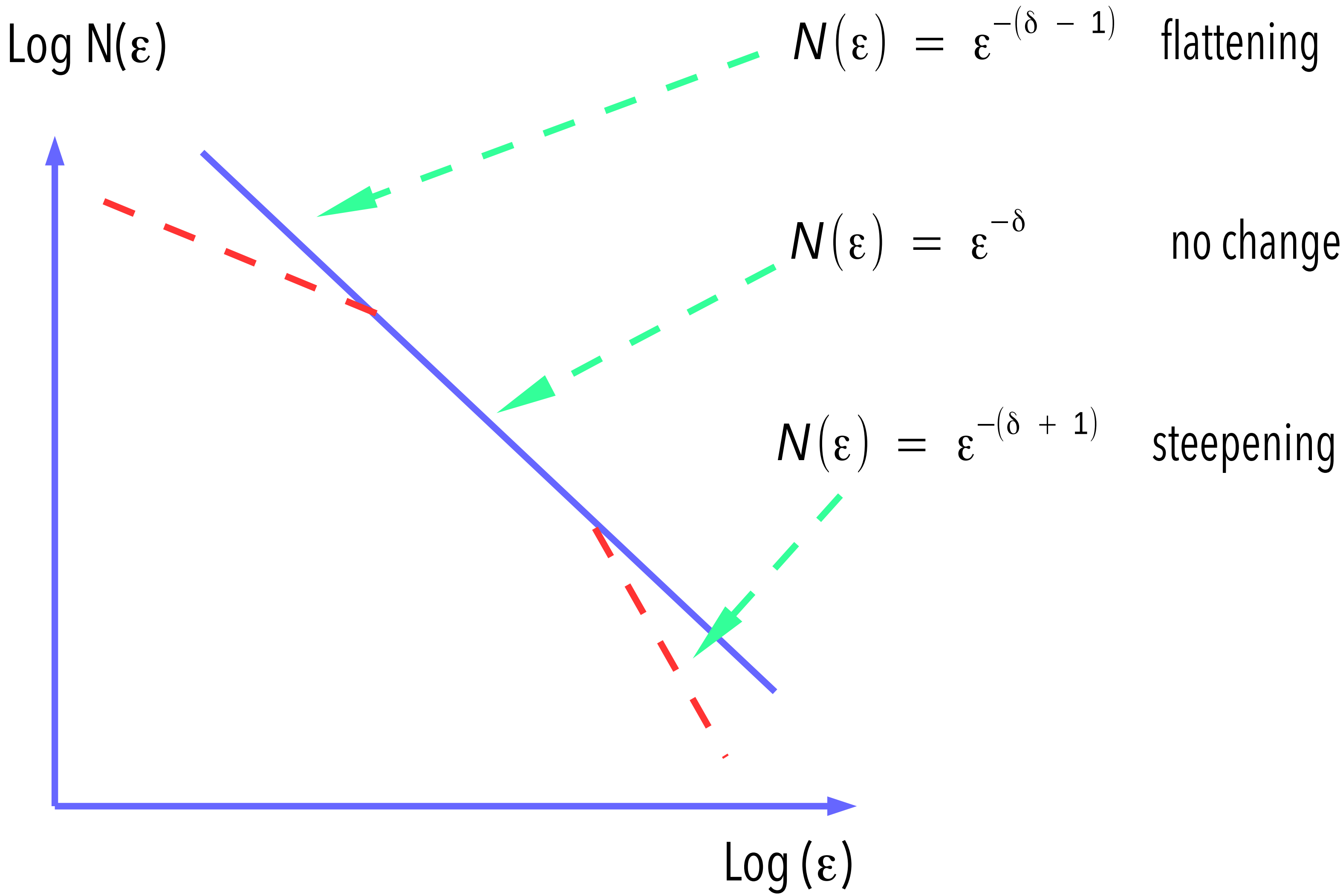
Stationary solutions with energy losses:

Inoization ($\sim c_1$ prevails) → $N(\varepsilon) = \varepsilon^{-(\delta - 1)}$ flattening

R.B. & A.E. ($\sim c_2$ prevails) → $N(\varepsilon) = \varepsilon^{-\delta}$ no change

Synchr. & I.C. ($\sim c_3$ prevails) → $N(\varepsilon) = \varepsilon^{-(\delta + 1)}$ steepening

Energy losses:



N.B. The emission spectrum changes accordingly [$S(\nu) \sim \nu^{2\alpha}$; $\alpha = (\delta - 1)/2$]

Synchrotron and Inverse Compton losses: $\frac{d\varepsilon}{dt} = b(H^2+H^2_{CMB})\varepsilon^2$

$$\frac{\partial N(\varepsilon,t)}{\partial t} + \frac{\partial}{\partial \varepsilon}\left(\frac{d\varepsilon}{dt} N(\varepsilon,t)\right) + \frac{N(\varepsilon,t)}{T_{conf}} = Q(\varepsilon,t)$$

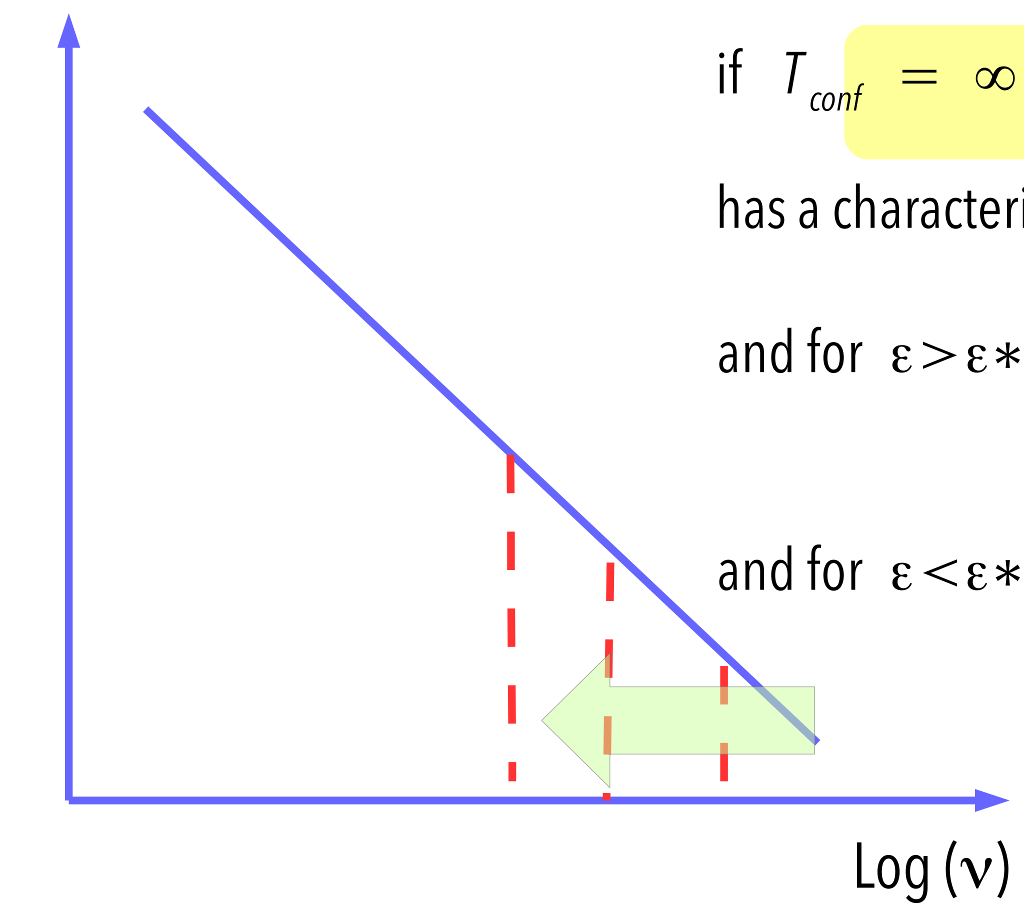
if $T_{conf} = \infty$; $Q(\varepsilon,t)=0$

has a characteristic energy $\varepsilon*(t) = \frac{1}{b(H^2+H^2_{CMB})t}$

and for $\varepsilon>\varepsilon* \rightarrow N(\varepsilon,t) = 0$

and for $\varepsilon<\varepsilon* \rightarrow N(\varepsilon,t) = \frac{N_o\varepsilon^{-\delta}}{[1-b(H^2+H^2_{CMB})\varepsilon t]^{2-\delta}}$

$$= \frac{N_o\varepsilon^{-\delta}}{(1-\varepsilon/\varepsilon*)^{2-\delta}}$$



The emission spectrum changes accordingly [$S(\nu) \sim \nu^{2\alpha}$; $\alpha = (\delta-1)/2$] : CUT-OFF

Synchrotron and Inverse Compton losses:

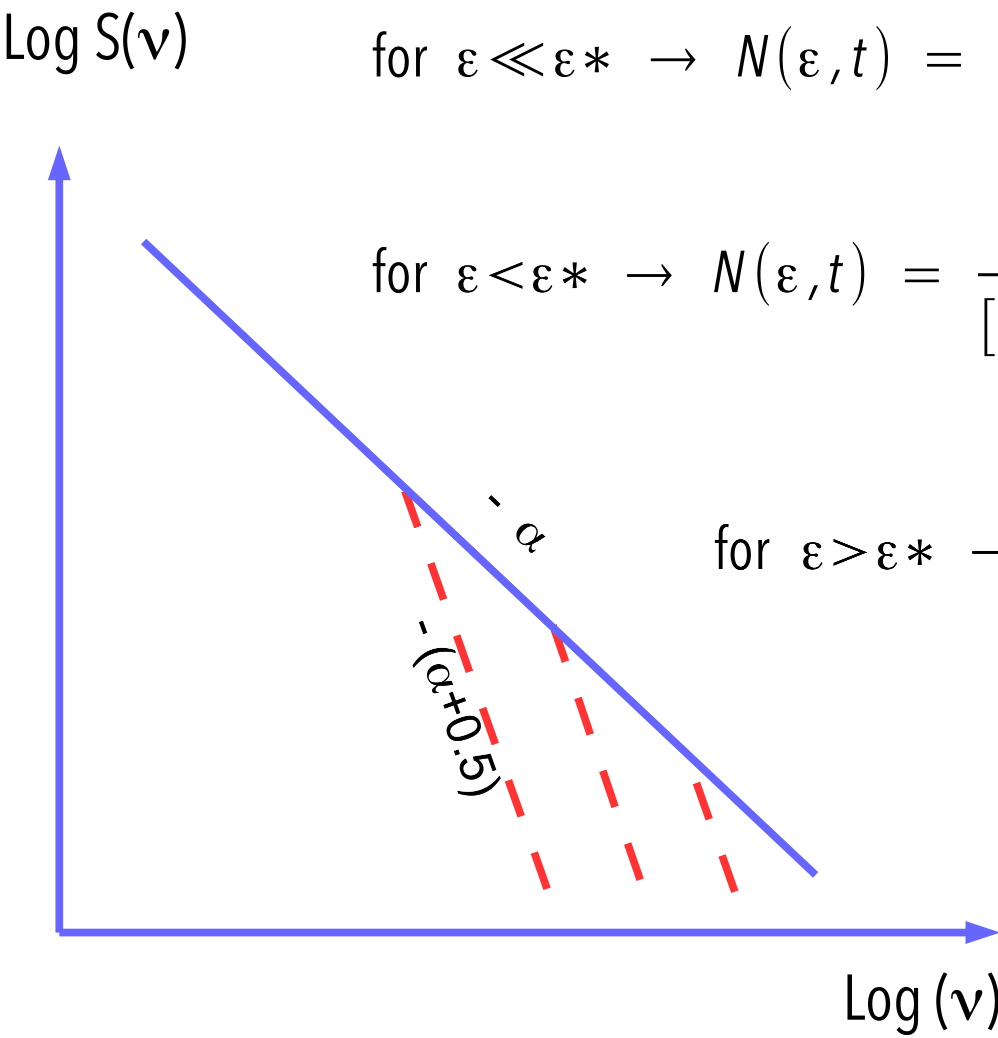
if $T_{conf} = \infty \;; \quad Q(\varepsilon,t) = A\varepsilon^{-\delta}$

the solution is:

for $\varepsilon \ll \varepsilon* \rightarrow N(\varepsilon,t) = A\varepsilon^{-\delta} t$

for $\varepsilon < \varepsilon* \rightarrow N(\varepsilon,t) = \frac{A\varepsilon^{-(\delta+1)}}{[(\delta-1)b(H^2+H^2_{CMB})]}\left[1-\left(1-b(H^2+H^2_{CMB})\right)\right]^{\delta-}$

for $\varepsilon > \varepsilon* \rightarrow N(\varepsilon,t) = \frac{A\varepsilon^{-\delta+1}}{(\delta-1)b(H^2+H^2_{CMB})}$



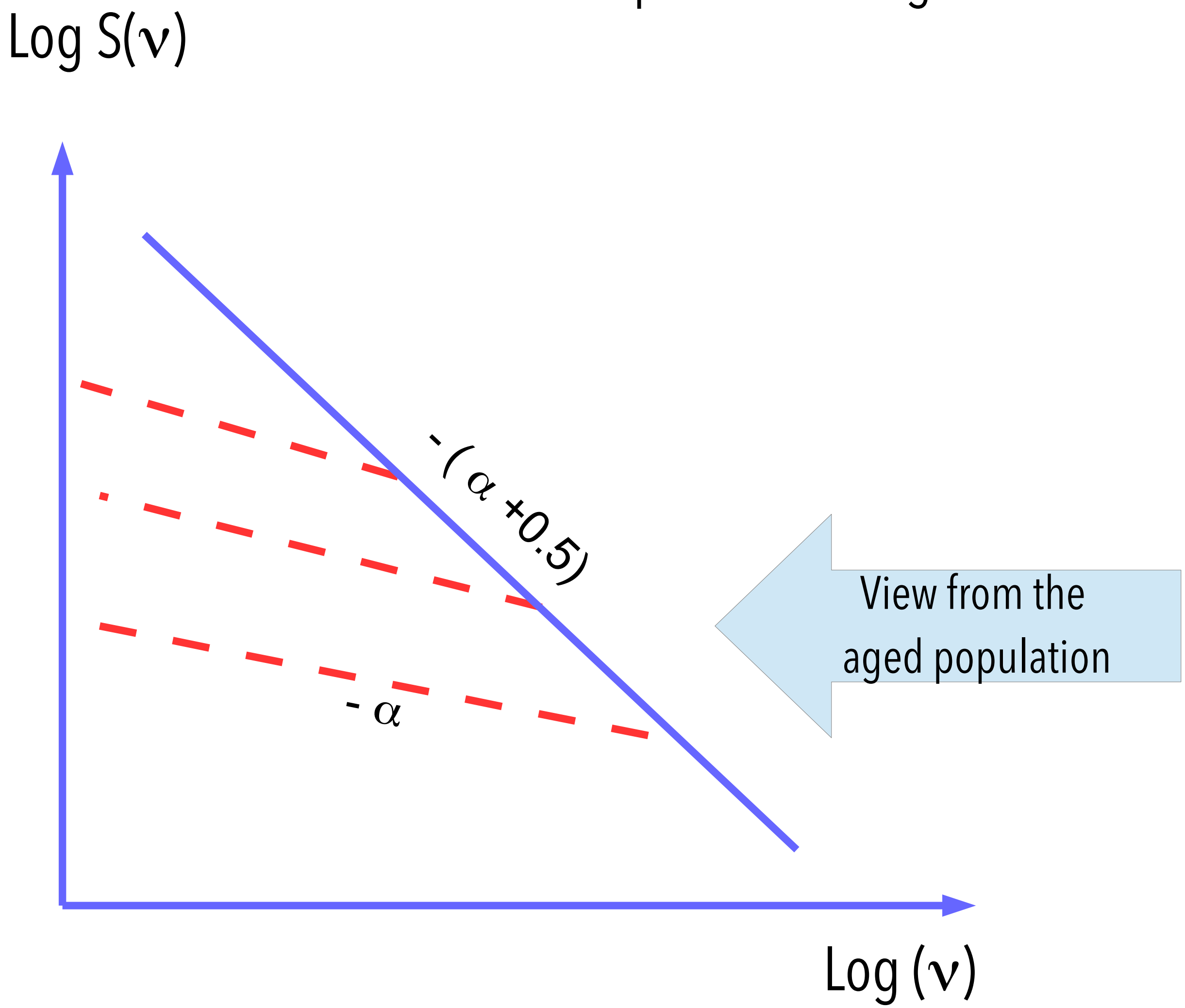
The emission spectrum changes accordingly [S (ν) ~ ν $^{2\alpha}$; α = (δ –1)/2] : BREAK

Summary of Synchrotron emission mechanism

Synchrotron and Inverse Compton losses:

if $Q(\varepsilon,t)=A\varepsilon^{-\delta}$ dominant over $N(\varepsilon,t) = N_o\varepsilon^{-\delta}$

the solution spectrum changes w.r.t the earlier example



The emission spectrum changes accordingly [$S(\nu) \sim \nu^{2\alpha}$; $\alpha = (\delta - 1)/2$] : BREAK

Summary of Synchrotron emission mechanism

- **POWER – LAW EMISSION SPECTRUM** $S(\nu) \sim \nu^{2\alpha}$ $[\ \alpha = (\delta - 1)/2]$

- Spectral profile modified by
  - Ageing
  - Self-absorption

The magnetic field topology determines the emission of individual relativistic electrons (oscillating perpendicularly to the local field direction) implying

- **LINEAR POLARIZATION**

up to a maximum fractional limit of 0.7 in optically thin radio emission

$$\left[P_{int}(\delta)\right]_{opt.thin} = \frac{P}{I} = \frac{3\delta+3}{3\delta+7}$$

Example of LINEAR POLARIZATION

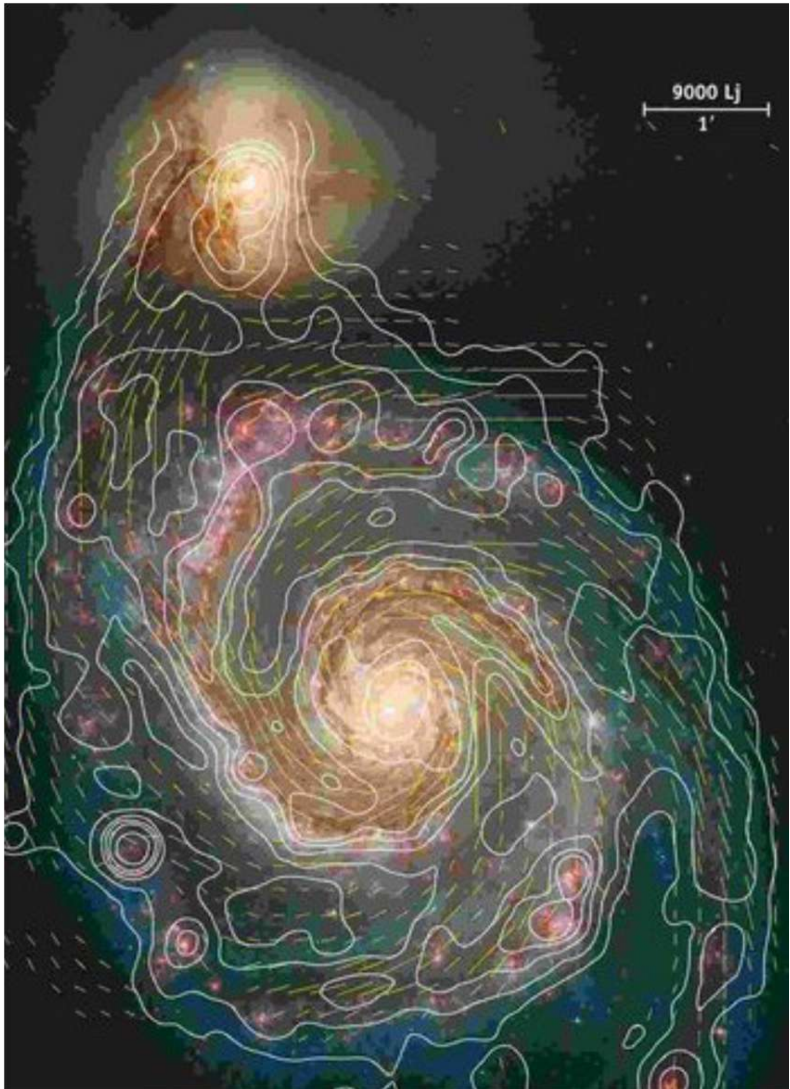
Optically thick regions select the field component emerging from the plasma cloud

$$\left[P_{\text{int}}(\delta)\right]_{opt.thick} = \frac{P}{I} = \frac{3}{16\delta+13}$$

If structured field topology, then

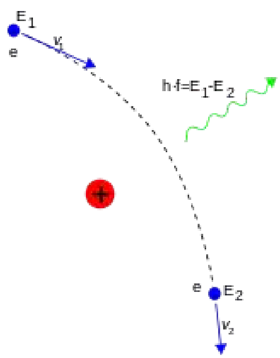
$$\left[\ P\ \right]_{obs} = P_{\text{int}} \frac{H_o^{\ 2}}{H_o^2+H_r^2}$$

2-D observation of a 3-D phenomenon



Summary of thermal Bremsstrahlung emission mechanism

Thermal bremsstrahlung is the main cooling mechanism of a hot, rarefied thermal plasma, via Coulomb interactions.



$$-\left(\frac{d\varepsilon}{dt}\right)_{br} = \frac{2}{3}\frac{q^2}{c^3}\,a^2 \qquad \text{where} \qquad a = \frac{q_1 q_2}{m\,x^2}$$

$$-\left(\frac{d\varepsilon}{dt}\right)_{br} = \frac{2}{3}\frac{Z^2 e^6}{c^3 m_e^2 b^4} \qquad \text{short range/time interaction} \qquad \Delta t \simeq \frac{2b}{v}$$

nearly flat frequency distribution of emitted energy. In case of a distribution of electrons n_e moving at v wrt a distribution of ions n_z, integrating over impact parameters b we get the specific emissivity

$$J_{br}(v,\nu) = \frac{32\pi e^6}{3c^3 m_e^2}\frac{1}{v} n_e n_z Z^2 \int_{b_{min}}^{b_{max}} \frac{db}{b} = \frac{32\pi e^6}{3c^3 m_e^2}\frac{1}{\boldsymbol{v}} \boldsymbol{n_e n_z} Z^2 \ln\left(\frac{b_{max}}{b_{min}}\right)$$

Unrealistic physical plasma, need to consider an equilibrium condition like a thermal plasma, i.e. Maxwell-Boltzmann distribution holds

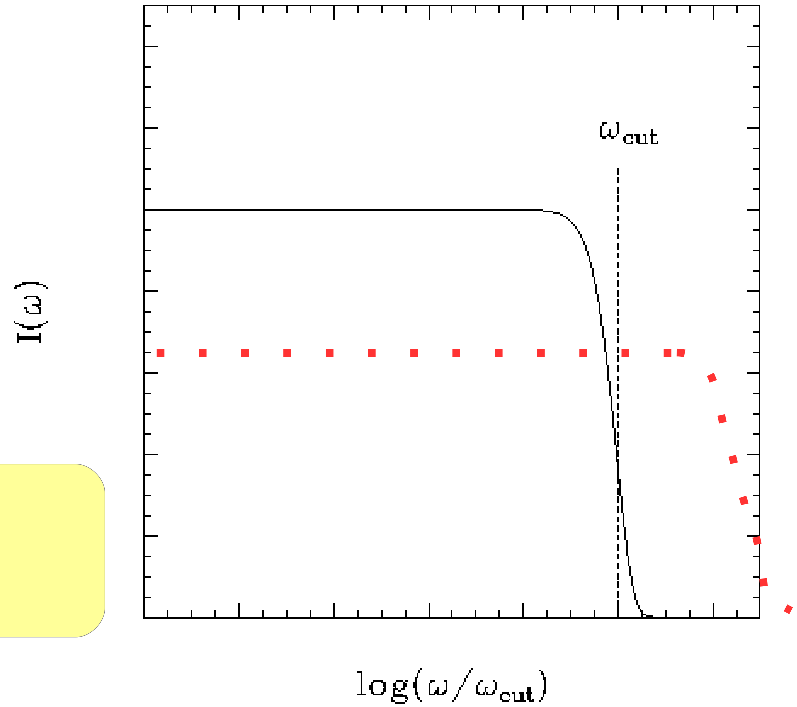
$$f(v)dv = 4\pi\left(\frac{m_e}{2\pi kT}\right)^{3/2} e^{-m_e v^2/2kT} v^2 dv$$

$n_e(v) = n_e f(v) dv$ replaces n_e in $J_{br}(v,\nu)$ which becomes dependent on T:

$$J_{br}(\nu,T) = \int_{v_{min}}^{\infty} J_{br}(\nu,v) f(v) dv = 6.8\times 10^{-38} \mathbf{T^{-1/2}} \mathbf{e^{-h\nu/kT}} \mathbf{n_e n_z} Z^2 g_{ff}(\nu,T) [\text{erg cm}^{-3} \text{Hz}^{-1}]$$

Lead to specific emissivity typical of the radio domain characteristic of warm and dense ionized plasmas (HII).

Hot and sparse plasmas (HIM, Galaxy Clusters) emit less photons but reach higher frequencies



Relativistic bremsstrahlung (summary)

minor remark: the Gaunt Factor is slightly different wrt the case of "thermal br."

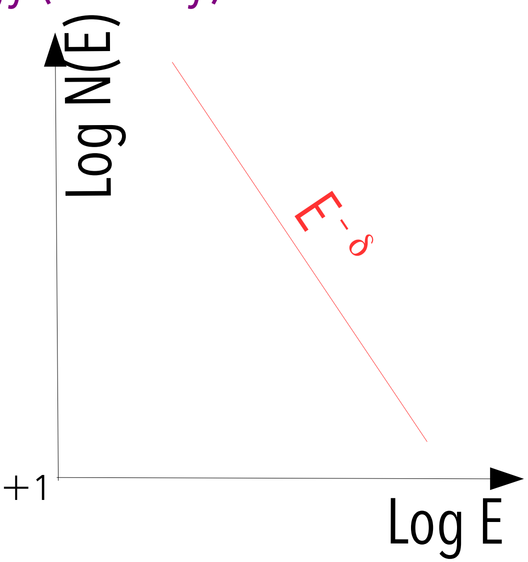
The emissivity for relativistic bremsstrahlung as a function of ν for a given velocity v is:

$$j_{rel,br}(v,\nu) = \frac{32\pi}{3}\frac{e^6}{m_e^2 c^3}\frac{1}{v} n_e n_Z Z^2 \ln\left(\frac{183}{Z^{1/3}}\right)$$

N.B. T is not a relevant concept any more, v must be used

The typical velocities of the particles are now relativistic and the energy (velocity) distribution is described by a power law:

$$n_e(E) \approx n_{e,o} E^{-\delta}$$

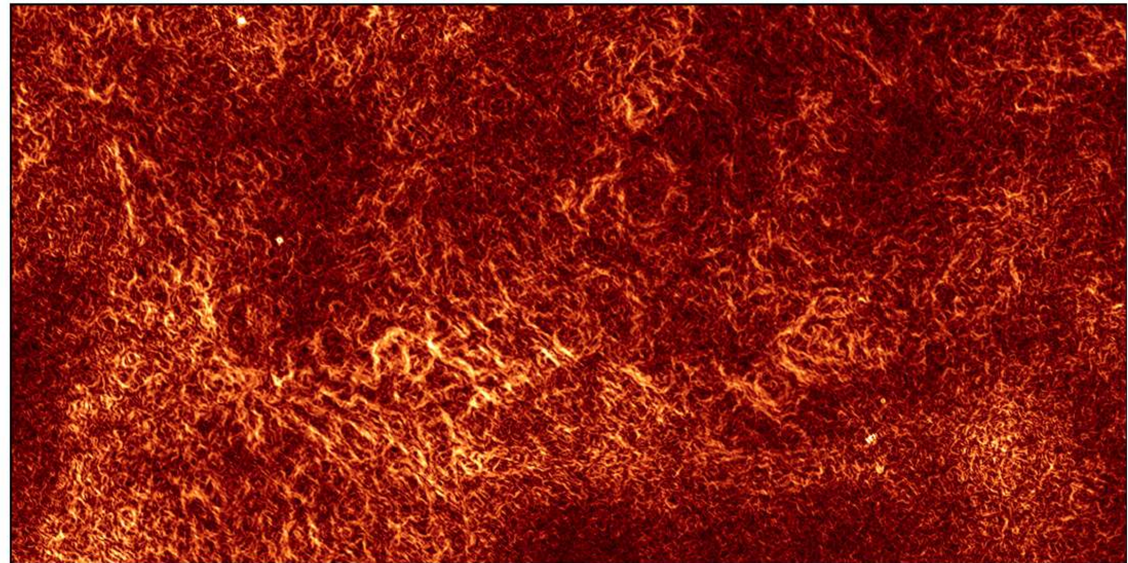


Integrating over velocities (energies)

$$j_{rel,br}(\nu) \approx \int_{h\nu}^{\infty} n_e(E) n_Z Z^2 dE \approx \int_{h\nu}^{\infty} E^{-\delta} dE \simeq \frac{E^{-\delta+1}}{1-\delta} \approx \nu^{-\delta+1}$$

the emitted spectrum is a power – law !

Plasma physics and the relevance of astrophysical magnetic fields



The 'snakes' are regions of gas where the density and magnetic field are changing rapidly as a result of turbulence. [Technical note: the image shows the gradient of linear polarisation over an 18-square-degree region of the Southern Galactic Plane. – Image credit – B. Gaensler et al. Data: CSIRO/ATCA

Motivation:

The Cosmic Magnetism is one of the Science Key Projects of the SKA telescope(s)

https://www.skatelescope.org/magnetism/

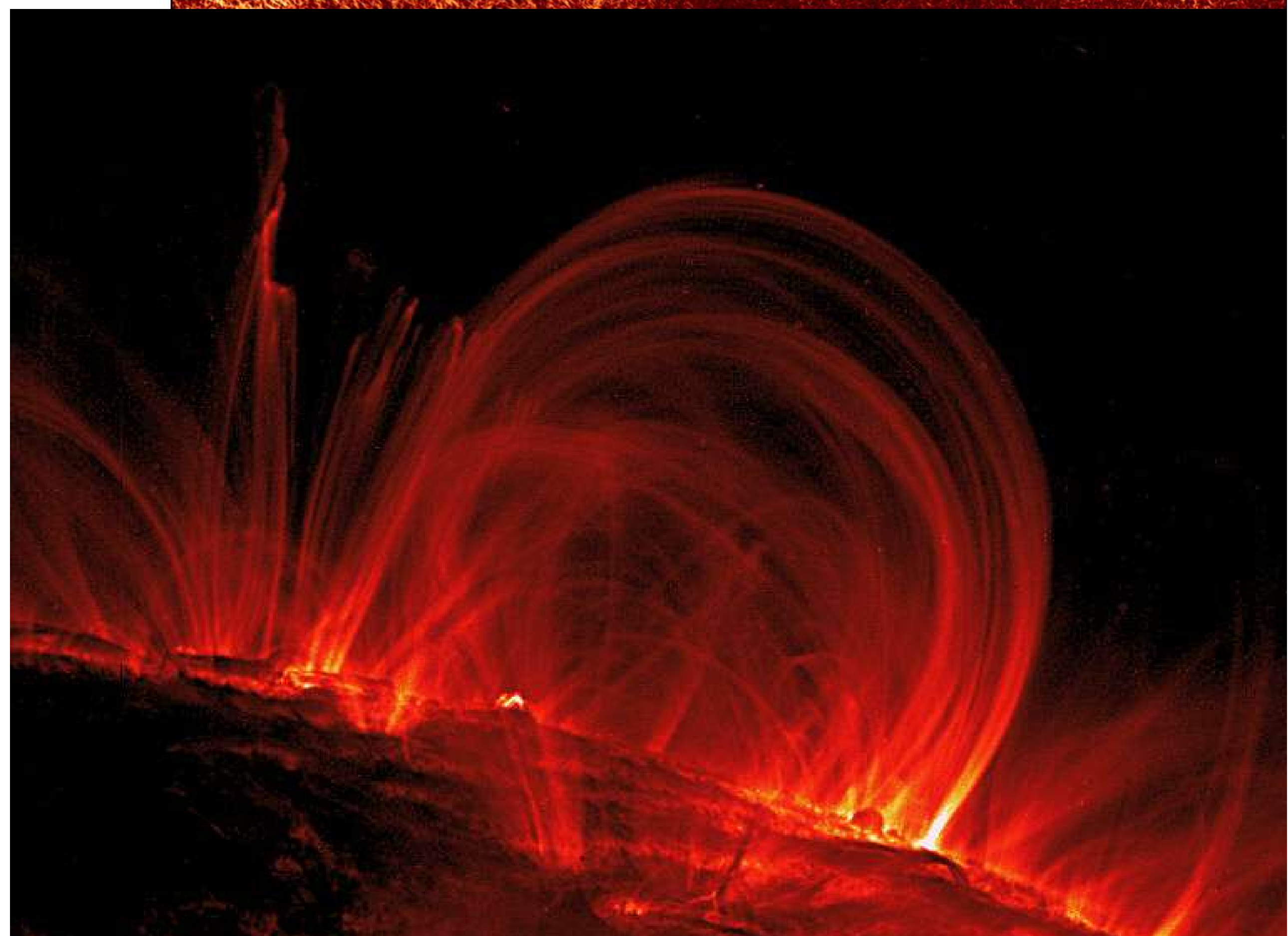


Image of the Sun's corona, taken in Nov 1999 by the Transition Region and Coronal Explorer (TRACE) satellite. The giant loops of gas seen arching above the Sun's surface delineate the patterns made by invisible magnetic fields

An astrophysical plasma isnot perfectly transparent to radiation: free electrons interact with e-m waves; the dielectric "constant" is defined as

number density of free / bound electrons

$$\varepsilon = 1 - \frac{4\pi e^2}{m_e}\left(\frac{n_e}{(\omega^2-\omega_o^2)} + \Sigma_i \frac{N_i}{(\omega^2-\omega_i^2)}\right)$$

pulsation of free (o) / bound (i) electrons

in the radio domain $\omega \ll \omega_i$, and $\omega_o = 0$ then $\varepsilon = 1 - \frac{4\pi e^2}{m_e}\frac{n_e}{\omega^2}$

The refraction index n_r: $n_r = \frac{c}{v_{gr}} = \sqrt{\varepsilon} = \sqrt{1 - \frac{4\pi e^2}{m_e}\frac{n_e}{\omega^2}} = \sqrt{1 - \frac{\nu_p^2}{\nu^2}}$

$$\nu_p = \sqrt{\frac{n_e e^2}{\pi m_e}} \simeq 9.1\times10^3\sqrt{n_e}\ \text{Hz}$$

Phase velocity $v_{ph} = \frac{c}{n_r} > c$ Group velocity $v_{gr} = c\cdot n_r < c$

Different frequencies travel with different velocities, a **delay** is introduced

1. Dispersion measure

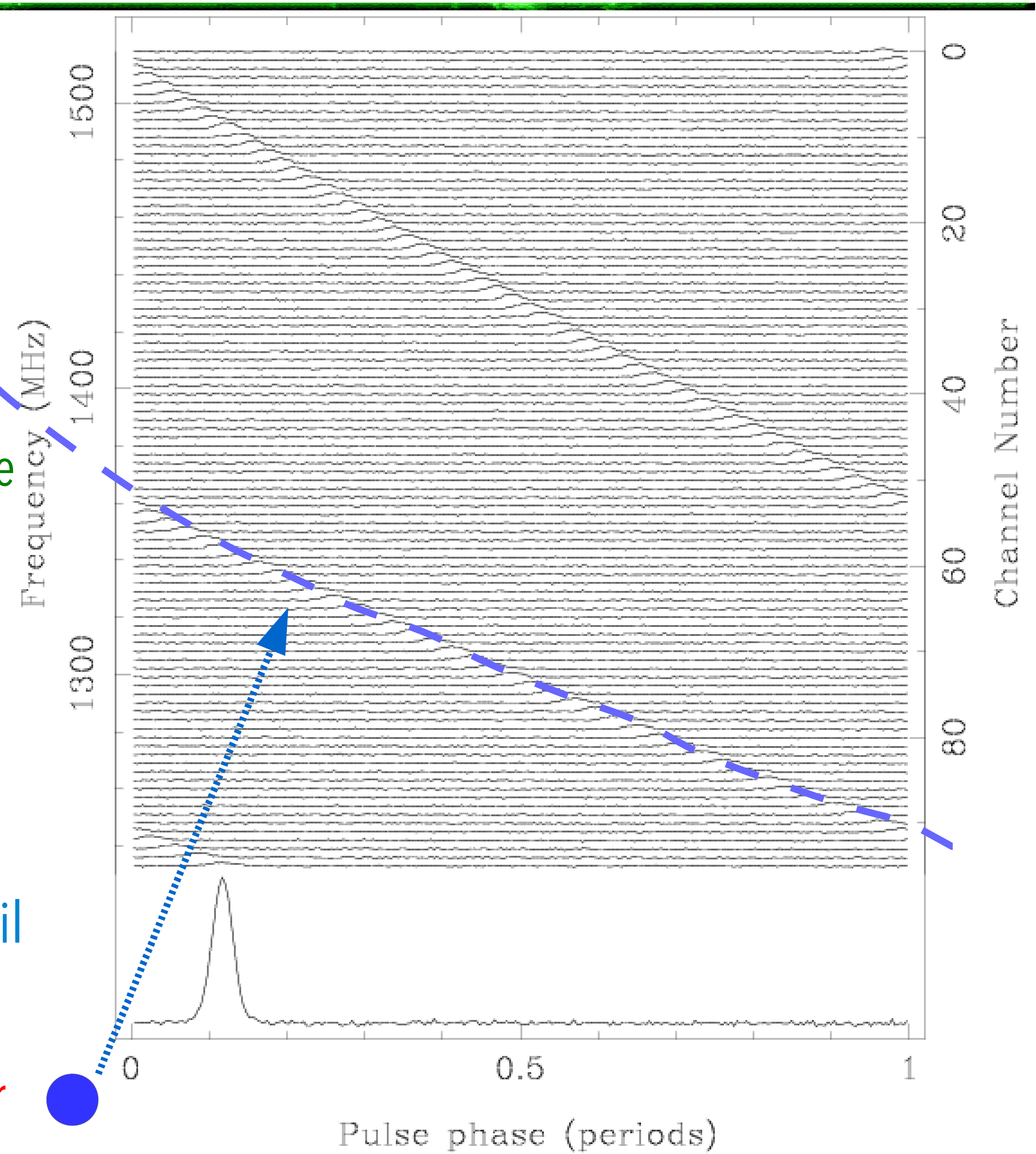
$$\Delta t = D.M. \frac{e^2}{2\pi m_e^2}\left(\frac{1}{\nu_1^2}-\frac{1}{\nu_2^2}\right)$$

where $$D.M. = \int_L n_e \, dl$$

A delay Δt is introduced in the arrival time

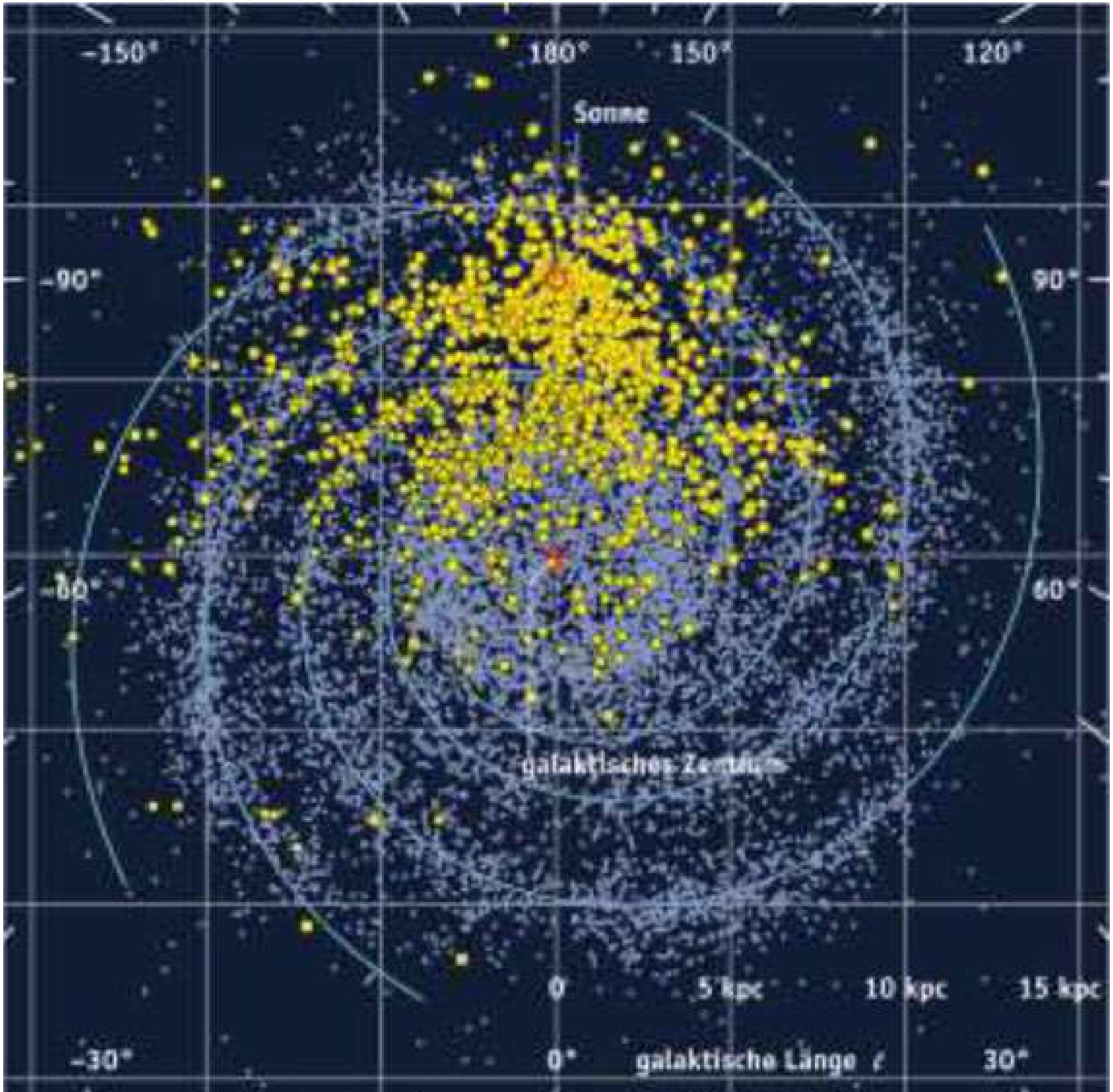
No free electrons $\Rightarrow$ DM = 0
Same arrival time for the pulses at any frequency

High n_e $\Rightarrow$ progressively larger Δt, until the delay can not be observed anymore
$\Rightarrow$ Slopes become steeped and steeper



$\Rightarrow$ Most pulsars observed in the solar neighborhood, no one in the galactic centre

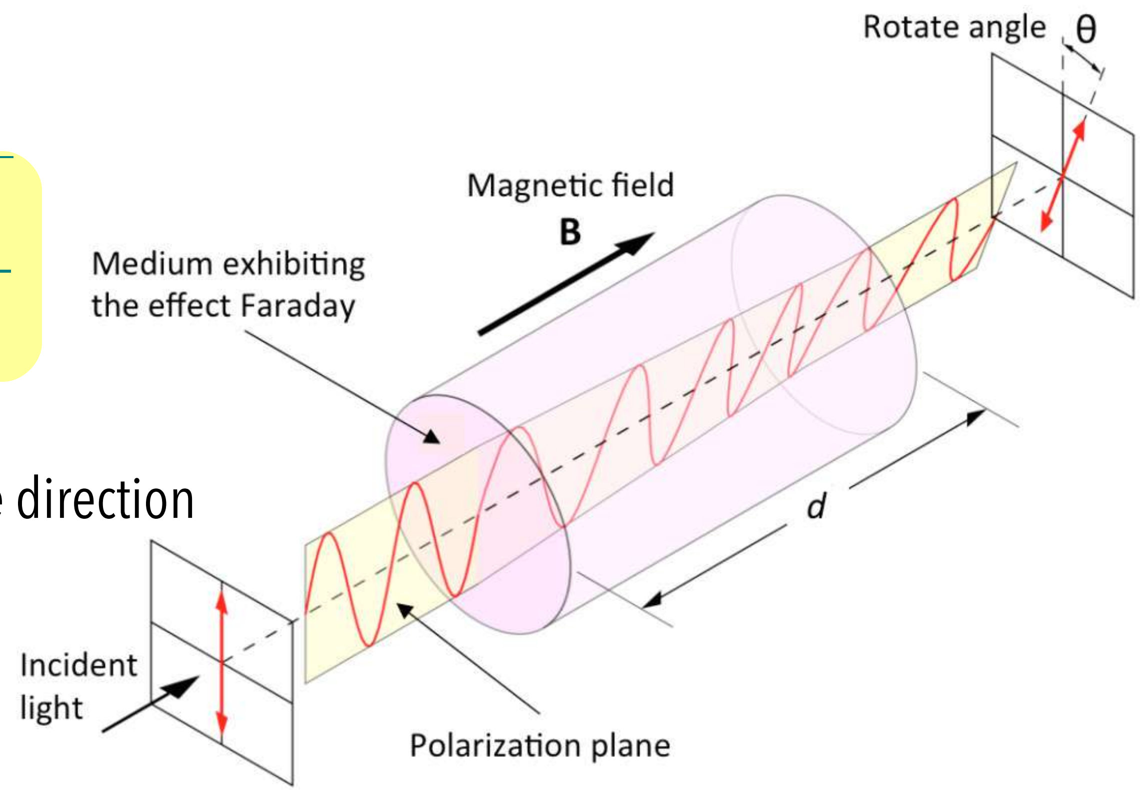
1. Pulsars in the Milky Way



2. Rotation measure

It affects **linearly polarized radiation**

$$n_{r(R,L)} = \sqrt{1 - \frac{\nu_p^2}{\nu^2} \frac{1}{1 \pm (\nu_L/\nu)\cos\theta}}$$



θ is the angle between the local $\vec{H}$ and the direction of propagation of the e-m wave

n_r is different for the two components of the electric polarization vector

The final, observed angle of linearly polarized emission is rotated by $\Delta\Phi = \lambda^2 \mathrm{R.M.}$

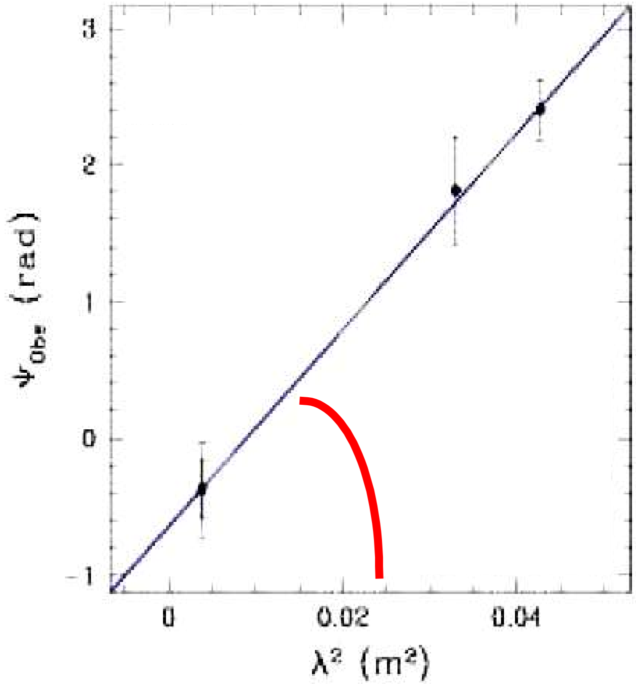
where $$\mathrm{R.M.} = \frac{2\pi e^3}{m_e^2 c^2} \int_L n_e H_{||} dl$$ is the **Rotation measure** $[\mathrm{rad\,m^{-2}}]$

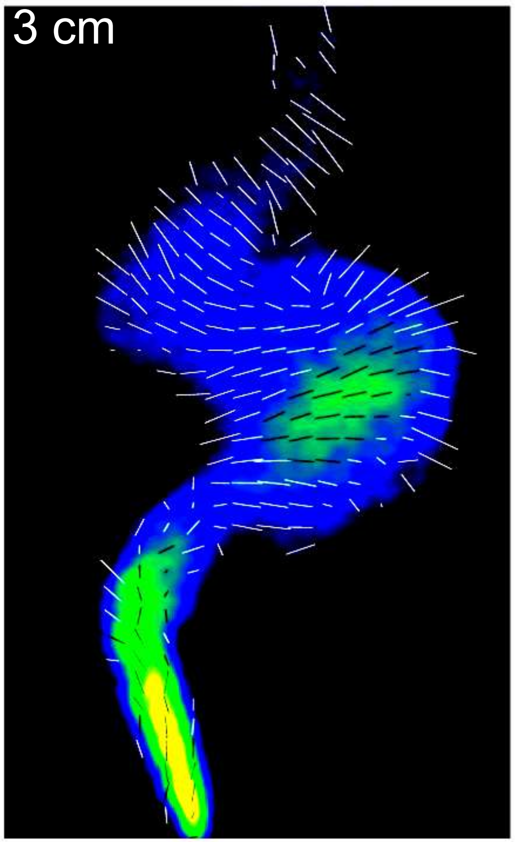
Radiotelescopes can measure $\chi_{obs}(\lambda) = \chi_o + \mathrm{R.M.} \cdot \lambda^2$

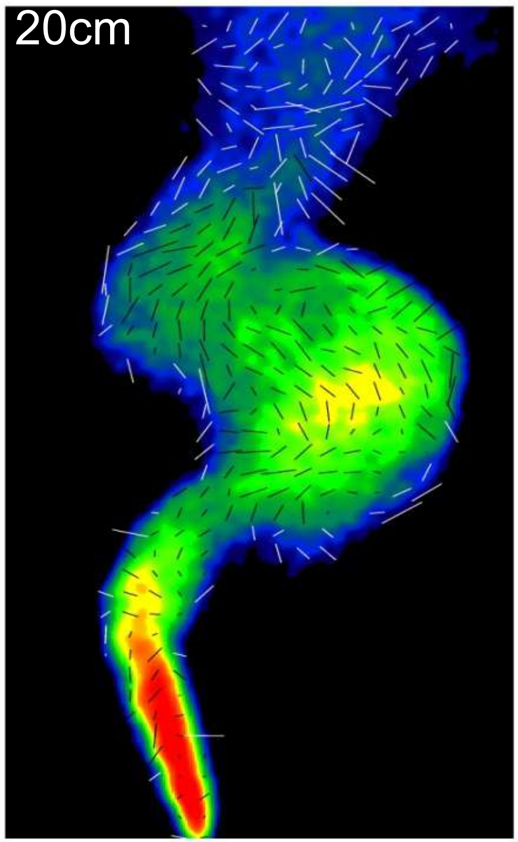
2. measuring the RM

$$\Delta\Phi = \lambda^2 R.M. \quad \text{where} \quad R.M. = \frac{2\pi e^3}{m_e^2 c^2}\int_L n_e H_{\parallel} dl$$

Many λ^2 need to be measured due to $2(n)\pi$ ambiguities





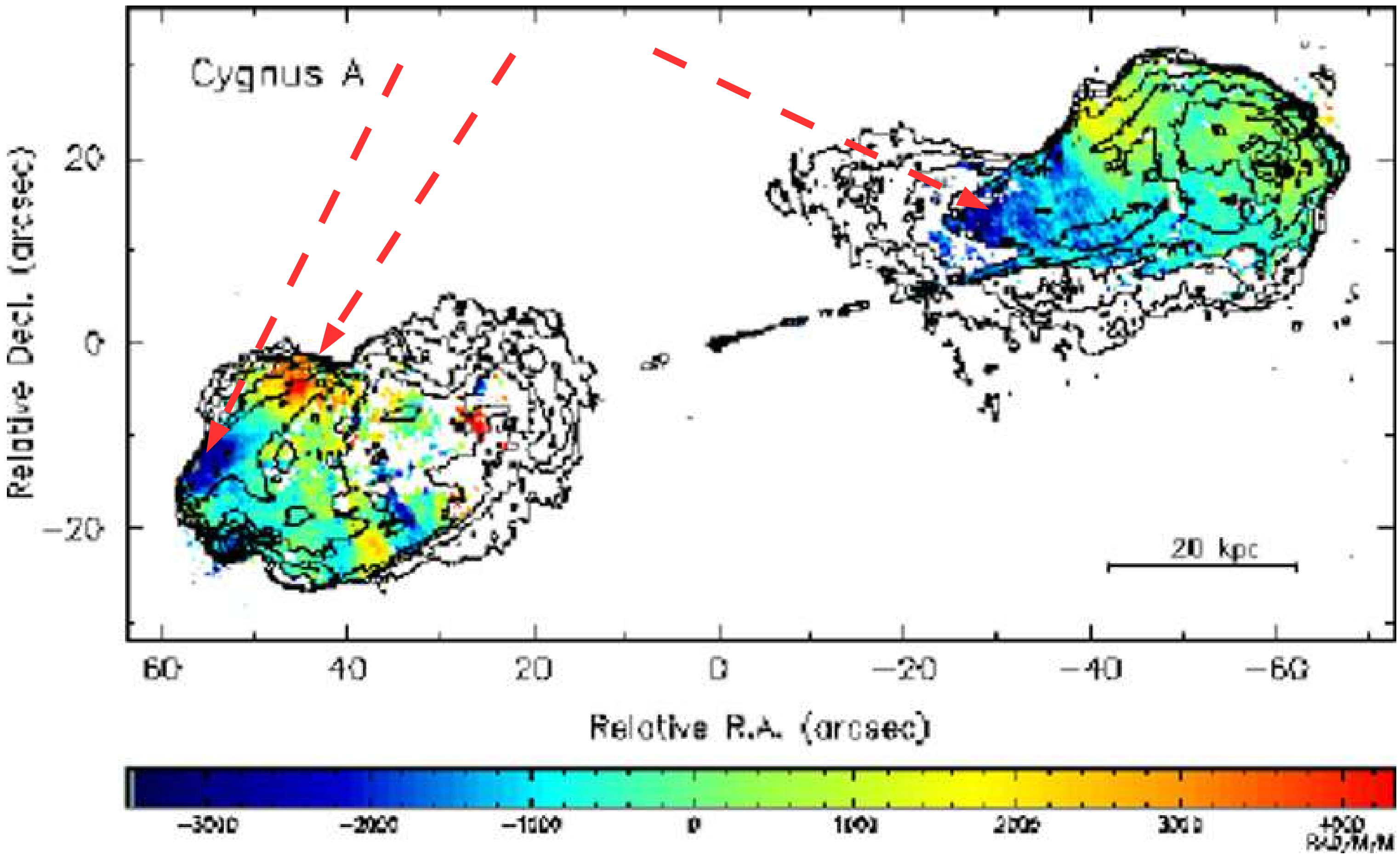


Locally / overall
different effects/rotations
can be generated

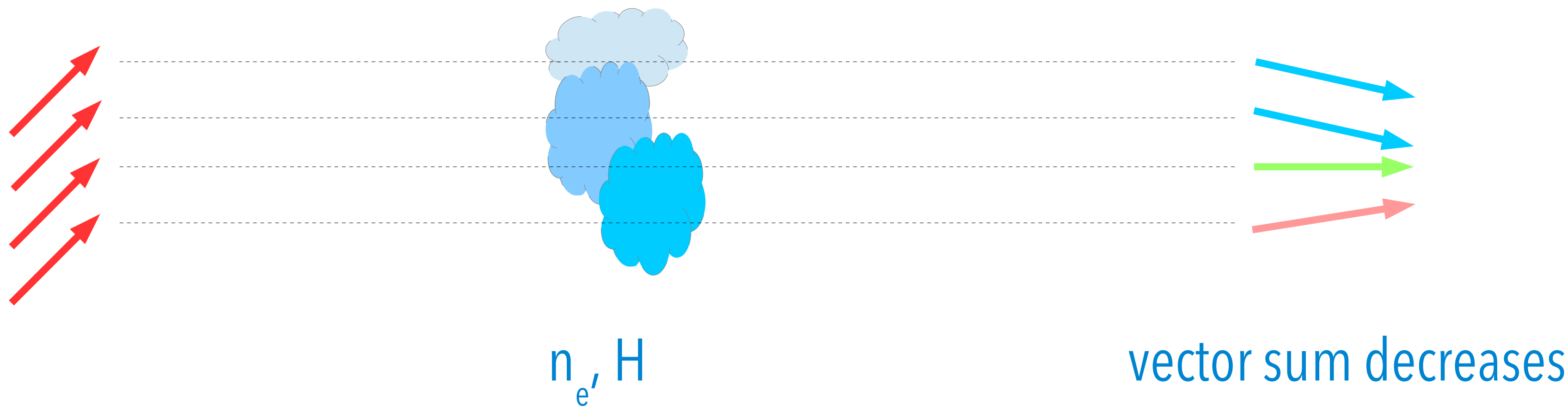
⇒ **Effect strictly connected to a given line of sight**

$$\Delta\Phi = \lambda^2 R.M. \quad \text{where} \quad R.M. = \frac{2\pi e^3}{m_e^2 c^2}\int_L n_e H_{\parallel} dl \quad [\mathrm{rad \cdot m^{-2}}]$$

Different colors → the B field component projected along the LoS changes sign!



2. Differential FR may depolarize (clouds with changing n_e, H along different LoS)



Rotation ~ λ^2, then sources at low frequencies are generally less polarized (depolarized) than at high frequencies

Depolarization:
$$DP = \frac{\text{fractional pol}(\nu_1)}{\text{fractional pol}(\nu_2)} \quad \text{with} \quad \nu_2 > \nu_1$$

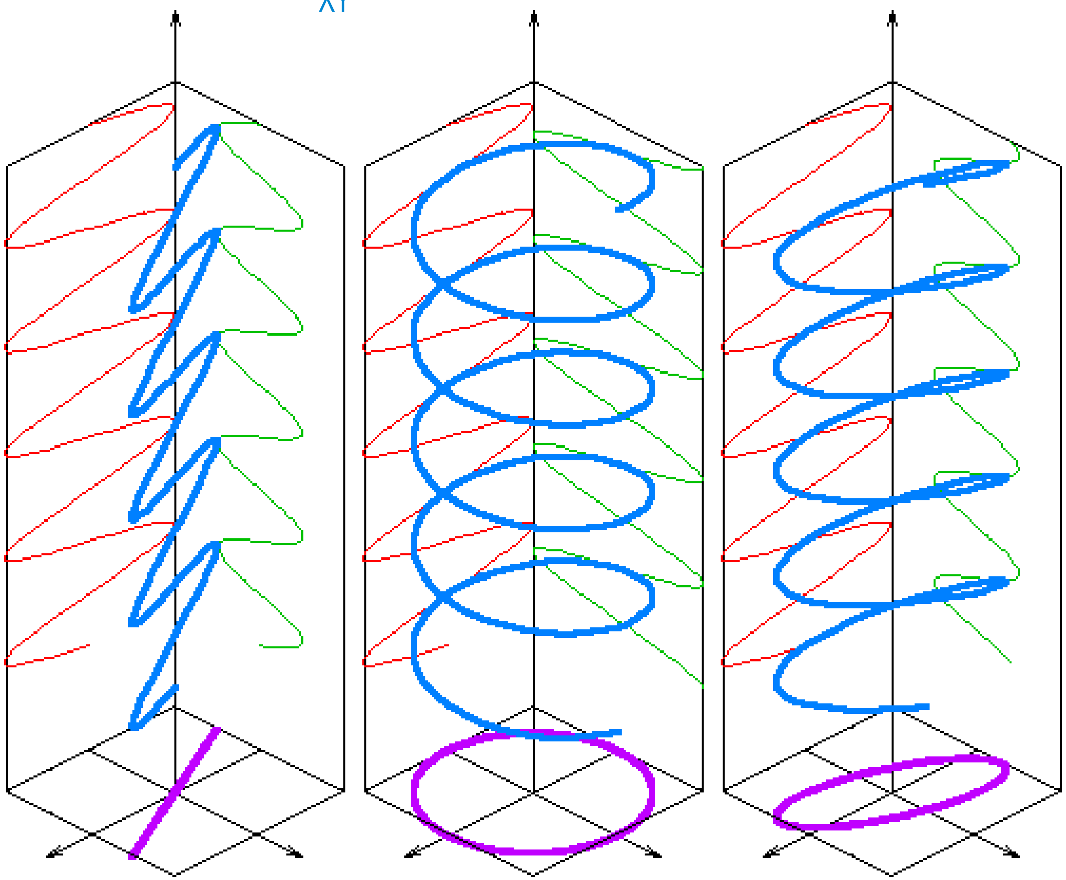
In general DP < 1, but there could be cases where it is slightly > 1 (spectrum!)

2. Linear polarization: $\delta_{XY} = 0^o$

   Circular polarization: $\delta_{XY} = 90^o$ E_X, E_Y are 180° out of phase

   Elliptical polarization: $0^o < \delta_{XY} < 180^o$, E rotates counter clockwise (right)

   $180^0 < \delta_{XY} < 360^o$, E rotates clockwise (left hand polariz.)



Watch: https://www.youtube.com/watch?v=Q0qrU4nprB0

https://www.youtube.com/watch?v=HH58VmUbOKM&ebc=ANyPxKph9jGXj29G6qK2lZVL5ZFKPqrjKD_1Wtc5tOwtHYHhkdMw9UJHKHcwuRUYDxBYCcT18waK