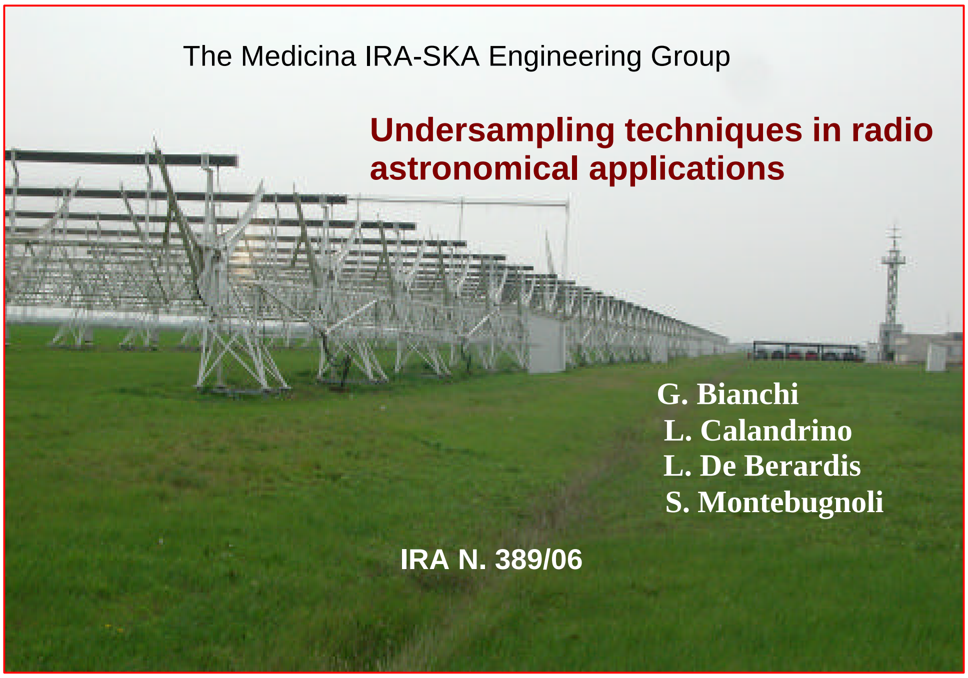

The Medicina IRA-SKA Engineering Group

Undersampling techniques in radio astronomical applications

G. Bianchi
L. Calandrino
L. De Berardis
S. Montebugnoli

IRA N. 389/06

Abstract

In the Medicina station laboratories we have done some tests about undersampling techniques to understand if this may be a good approach for BEST project [1]. First results seem to be very promising, but it is very important to have a particular care of clock source and clock distribution to ADC: it needs a low jitter clock to have a high SNR (Signal to Noise Ratio).

Introduction

By Shannon's theorem, an analog signal must be sampled at a rate of $f_s>2f_H$ to avoid the loss of information, where f_H is the maximum frequency of the signal and f_s is the sampling frequency. This condition is sufficient but, in general, not necessary. It becomes also necessary when the signal has a bandwidth from DC to f_H, while for band-pass signal the sampling frequency can be lower than $2f_H$. In this case the sampling condition is:

$$f_s>2B$$

where $B = f_H – f_L$ is the signal bandwidth and f_L is the minimum frequency of the signal. This technique is called undersampling.
This means that undersampling is possible if the clock frequency f_s is greater than 2B, but not all f_s values between 2B and $2f_H$ are permissible, because if $f_s<2f_H$, then a phenomena called aliasing could occur. To avoid aliasing, these equations have to be observed [2]:

$$n < \frac{f_L}{B}$$

$$\frac{2}{n+1} f_H < f_s < \frac{2}{n} f_L$$

where n is a positive integer. In this case we have n ranges of frequencies where it is possible undersampling; n=0 is the standard Nyquist sampling.
In real conditions, n cannot be too low, because we don't have an ideal filter at RF level and in this case it needs a sufficient gap between two adjacent bands. In any way, the RF filter has to be the most selective possible.
The undersampled band will be inverted or not inverted; it depends on the position of the RF band in comparison to the sampling frequency f_s. If n+1 is even, the undersampled band will be inverted, if n+1 is odd, the undersampled band will not be inverted.
The Northern Cross radiotelescope (figure 1) has got a 16MHz bandwidth cantered at 408 MHz. Using undersampling technique it is possible to sample this band with only 40 MSPS for example. In this case, with an only action, we convert down the signal and digitalize it and it doesn't need mixer and local oscillators. Therefore we reduce the cost and increase the reliability.

Fig. 1. The Northern Cross radiotelescope

Jitter considerations

Jitter is a low random variation on the clock period. In an ADC, this means to sample an input signal with a random clock period. Intuitively, since in the undersampling technique the input signals frequency is higher than sampling frequency, low oscillations of the sampling instants cause high voltage variations on the sampled signal. As shown in figure 2, when the signal slops is high, an error on the sampling instant causes a high error of the voltage signal.

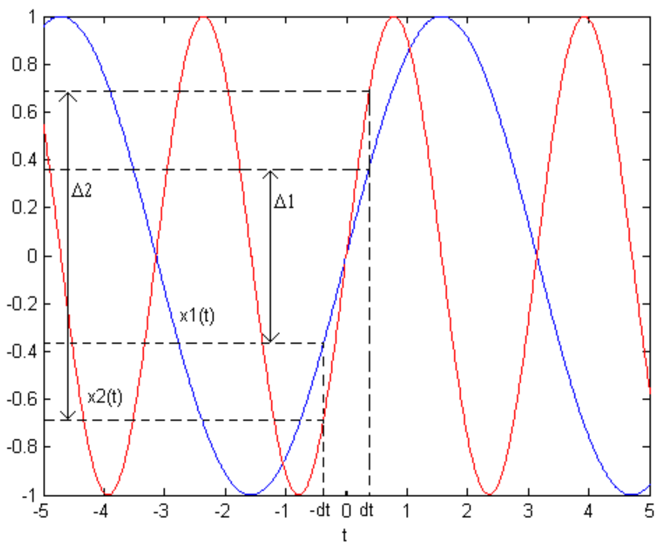

Fig. 2. Higher is the signal frequency, and higher is the error signal value due to jitter

Total jitter is composed by two factors (RSS: Root-Sum-Square): ADC aperture jitter and clock jitter [3]:

$$t_j = \sqrt{t_{jCLK}^2 + t_{jADC}^2}$$

Since aperture jitter depends on the ADC, it cannot be improved. Contrarily clock jitter depends on the clock source and clock distribution system and it can be improved.

Jitter causes degrade of the conversion performances; it means an introduction of phase noise in the frequency dominion and consequently a decrease of the SNR of the system. The SNR due to the jitter is [4]:

$$SNR_j = -20\log_{10}(2p\, f_{IN} t_j)$$

where:

$t_j = rms\ time\ jitter$

$f_{IN} = input\ frequency$

Results

We planned some tests with different input signal: first we have done tests with monochromatic signal, then we have used radio astronomical signal from BEST-1. In the second case, we used the block diagram reported in figure 3. In figure 4 is reported the test bench.

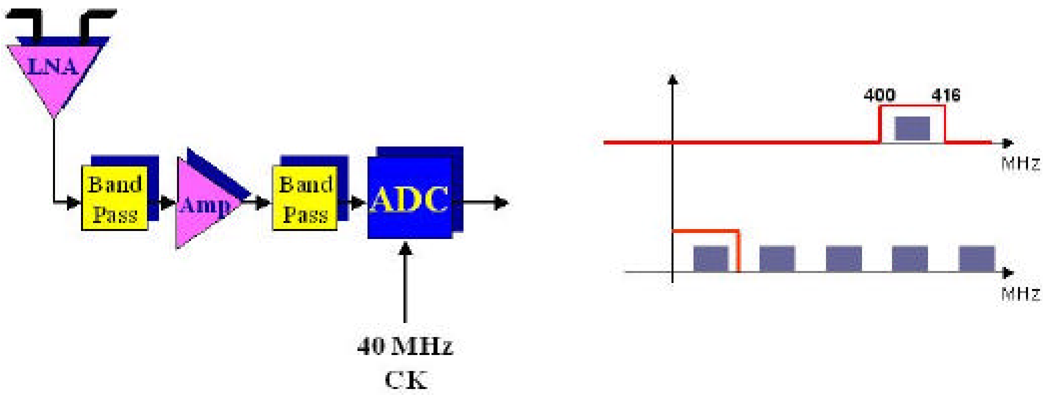

Fig. 3. Undersampling block diagram.

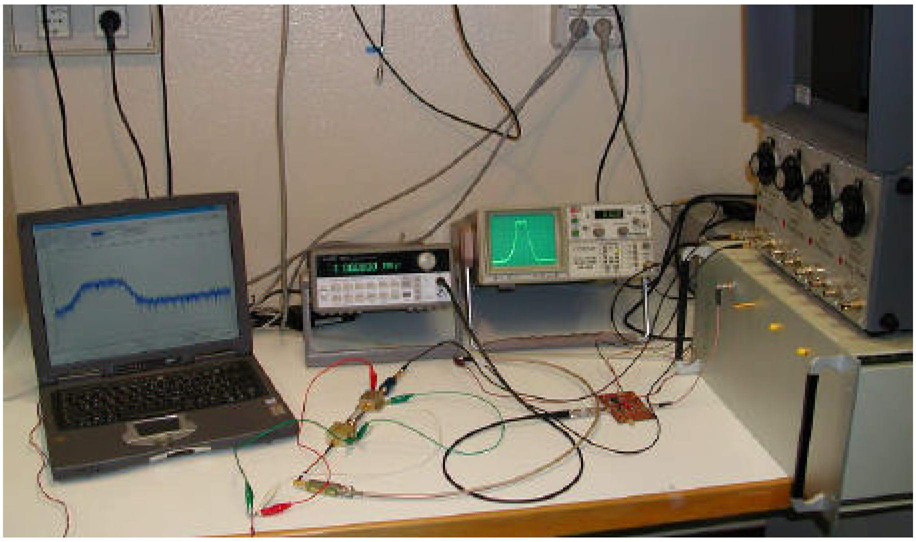

Fig. 4. Test bench.

Test with monochromatic signals

We have took two different input signals at 20 MHz and at 100 MHz and we have sampled them with two different clock sources at 80 MSPS:

1. Clock squared from signal generator
2. Clock from PLL

In the first case, moving the input frequency from 20 MHz to 100 MHz we have seen an increase of the power noise level of 10 dB as shown in figure 5.

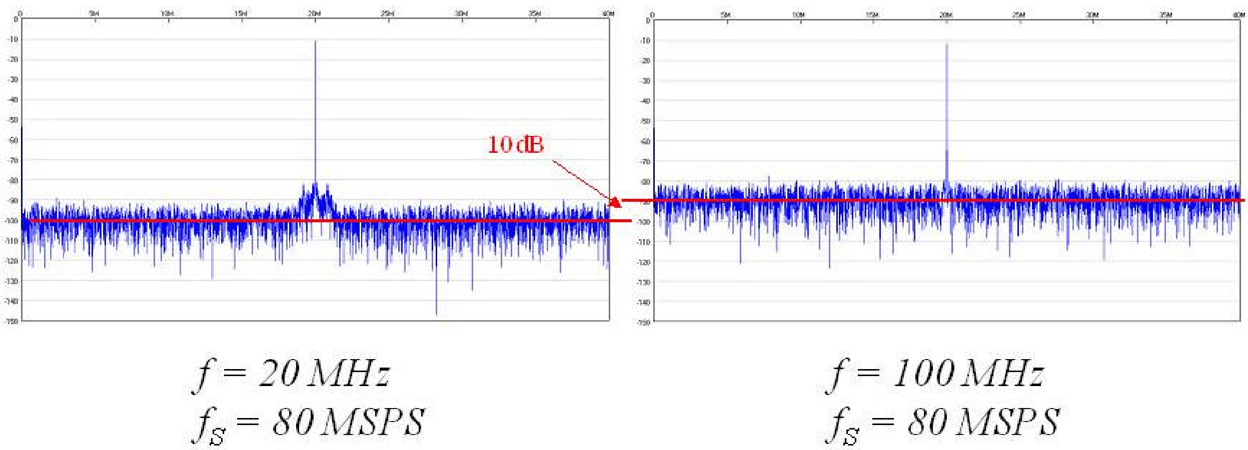

Fig. 5. Undersampling results with clock squared from signal generator.

In the second case (PLL source) when we have moved the input frequency from 20 MHz to 100 MHz, we have seen the same level of power noise, but we have also observed an increase of phase noise around the carrier due to PLL as shown in figure 6.

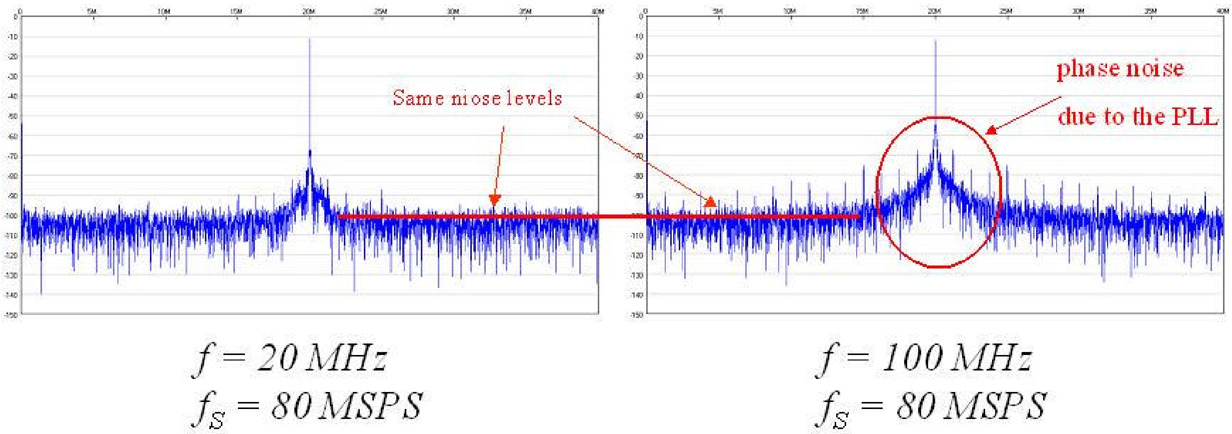

Fig. 6. *Undersampling results with clock from PLL.*

Test with radio astronomical signal from Northern Cross antenna (BEST-1)

We have sampled 8 MHz bandwidth centred at 408 MHz from BEST-1 with 80 MHz clock frequency. We can see the result in the figure 7. The noise level is –105dBm and it is evident the radio interference like in the spectrum analyser.

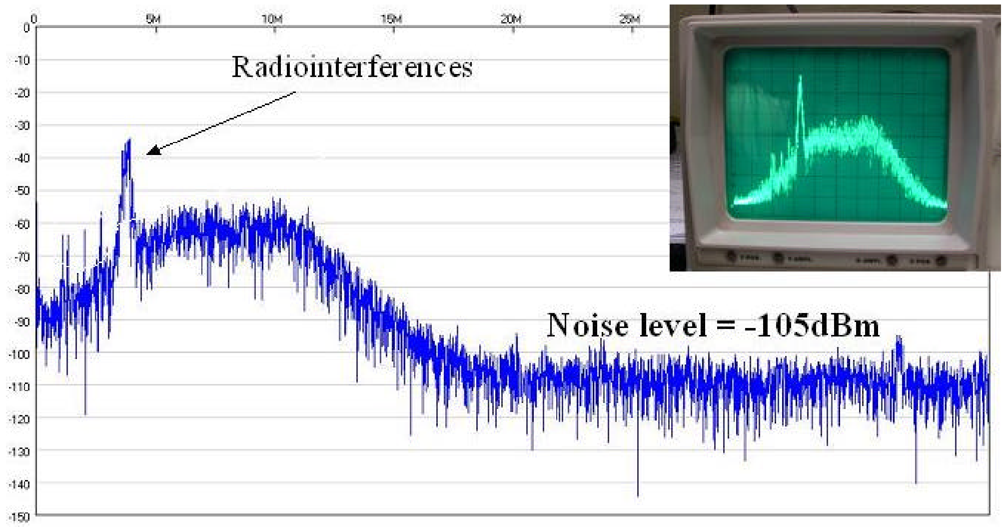

Fig. 7. Undersampling result with 80 MHz clock frequency.

If we decrease even more the clock sample (from 80 MSPS to 50 MSPS), we see a large increase of noise level (figure 8).

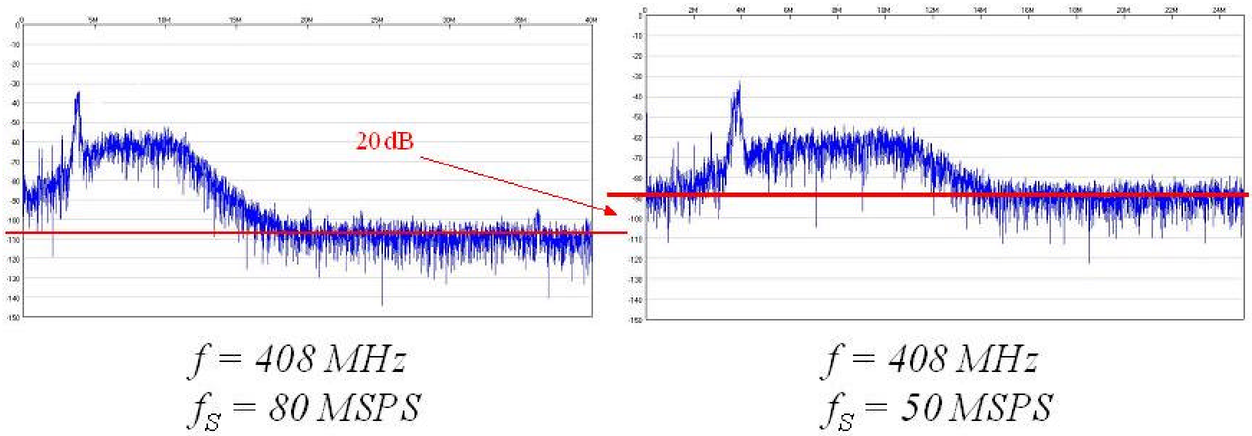

Fig. 8. Different results with 80 MHz and 50 MHz clock frequency.

For obvious reason, clock has to be very stable (very low jitter):

increasing of the clock jitter → increasing of the power floor.

This problem is very crucial for high undersampling factor ($f_{NYQUIST}/f_{SAMPLE}$).

Conclusions

Undersampling in radioastronomical applications seems to be possible, but it is very important to have a particular care of clock source and clock distribution to ADC: it needs a low jitter clock to have a high SNR. Now we are designing a new low jitter clock distribution system and then we are going to try it in BEST-1 and we have in mind to use undersampling in the BEST project.

More details about undersampling technique are reported in the references [5] – [9].

Appendix:

List of the commercial ADC suitable for undersampling

Company	Name	Symbol rate [MSPS]	Input bandwidth [MHz]	N bits	N channels
Analog Devices	AD9446-100	100	540	16	1
	AD9445-125	125	615	14	1
	AD9445-105	105	615	14	1
	AD9444	80	650	14	1
	AD9481	250	750	8	1
	AD9245	80	500	14	1
	AD9433-105	105	750	12	1
	AD9433-125	125	750	12	1
	AD9430	210	700	12	1
	AD9432	105	500	12	1
National Semiconductor	ADC08200	200	500	8	1
	ADCS9888	205	500	8	3
	ADC12L080	80	450	12	1
Linear Technology	LTC2224	135	775	12	1
	LTC2220-1	185	775	12	1
	LTC2294	80	575	12	2
	LTC2299	80	575	14	2
	LTC2208	130	700	16	1
	LTC2207	105	700	16	1
	LTC2206	80	700	16	1
	LTC2255	125	640	14	1
	LTC2253	125	640	12	1
Maxim	MAX100	250	1200	8	1
	MAX19542	170	900	12	1
	MAX12528	80	750	12	2
	MAX1215	250	700	12	1
	MAX1214	210	700	12	1
	MAX1213	170	700	12	2
	MAX1124	250	600	10	1
	MAX1123	210	600	10	1
	MAX1122	170	600	10	1
	MAX1121	250	600	8	1
	MAX19586	80	600	16	1
	MAX1219	210	800	12	2
	MAX1218	170	800	12	2
	MAX1217	125	800	12	2
Texas Instruments	ADS5424	105	570	14	1
	ADS5423	80	570	14	
	ADS5500	125	750	14	1
	ADS5541	105	750	14	1
	ADS5542	80	750	14	1
	ADS5520	125	750	12	1
	ADS5521	105	750	12	1
	ADS5522	80	750	12	1
	ADS5410	80	1000	12	2

References

[1] S. Montebugnoli, G. Bianchi, C. Bortolotti, A. Cattani, A. Cremonini, A. Maccaferri, F. Perini, M. Roma, J. Roda, P. Zacchiroli, "**Italian SKA test bed based on cylindrical antennas**", Astronomische Nachrichten, Volume 327, Issue 5-6 (p 624-625), 11 May 2006.

[2] Angelo Ricotta, "Considerazioni sulla digitalizzazione e l'elaborazione dei segnali SODAR"
Nota Interna, IFA-CNR, Luglio 1983

[3] When undersampling, clock jitter does matter
By Bonnie Baker -- EDN, 8/4/2005
www.edn.com/article/CA629307.html

[4] On the Jitter Requirements of the Sampling Clock for Analog-to-Digital Converters
Nicola Da Dalt, Moritz Harteneck, Christoph Sandner and Andreas Wiesbauer
IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—I: FUNDAMENTAL THEORY AND APPLICATIONS, VOL. 49, NO. 9, SEPTEMBER 2002

[5] Effect of Jitter on Asynchronous Sampling With Finite Number of Samples
Nicola Da Dalt
IEEE TRANSACTIONS ON CIRCUITS AND SYSTEMS—II: EXPRESS BRIEFS , VOL. 51, NO. 12, DECEMBER 2004

[6] Audio precision technote TN-23
Jitter theory
by Julian Dunn

[7] Measuring Spurious Free Dynamic Range in a D/A Converter
Intersil Technical Brief, January 1995 - TB326
Authors: Juan Garcia, Stephen G. LaJeunesse, Douglas Bartow

[8] Analog Devices Application Note AN-410
Overcoming Converter Nonlinearities with Dither
by Brad Brannon

[9] A High-Performance Digital-Transceiver Design, Part 1
by James Scarlett, KD7O