17 Partial differential equations

Functions:

sor: elliptic pde solved by the simultaneous overrelaxation method
adi: elliptic pde solved by the alternating direction implicit method

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sor

Description:

This function solves an elliptic partial differential equation on a rectangle by the simultaneous overrelaxation method. If u is the unknown solution then the discretized form of the equation for this method is

$$a_{j,l}u_{j+1,l} + b_{j,l}u_{j-1,l} + c_{j,l}u_{j,l+1} + d_{j,l}u_{j,l-1} + e_{j,l}u_{j,l} = f_{j,l}$$

sor(a, b, c, d, e, f, u, rjac) $\longrightarrow$ **u**

Where:

a is a real square matrix dimensioned to the grid size,
b is a real square matrix dimensioned to the grid size,
c is a real square matrix dimensioned to the grid size,
d is a real square matrix dimensioned to the grid size,
e is a real square matrix dimensioned to the grid size,
f is a real square matrix dimensioned to the grid size,
u is a real square matrix dimensioned to the grid sized representing on input an initial guess for the solution and on output the computed solution,
rjac is a real number being an approximation to the spectral radius of the Jacobi iteration.

Pre-variables:

sor_maxits [1000] the maximum allowed number of iterations,
sor_eps [1.0d-5] a floating point number representing the relative convergence factor.

Example:

In this example an elliptic partial differential equation is solved by the simultaneous overrelaxation method.

```
-> pi : float(%pi)?
3.14
-> -----------------------------------------------------------------------#
-> a : constant_matrix(1,11,11);
-> b : a; c : a; d : a;
-> e : constant_matrix(-4,11,11);
-> f : insert_elt(2,6,6,zero(11,11));
-> u : zero(11,11);
-> rjac : eval(float(cos(%pi/11)))?
0.96
-> sor(a,b,c,d,e,f,u,rjac)?
 0.00  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00 |
|                                                                          |
| 0.00  -0.02  -0.04  -0.06  -0.08  -0.09  -0.08  -0.06  -0.04  -0.02  0.00 |
|                                                                          |
| 0.00  -0.04  -0.09  -0.13  -0.17  -0.19  -0.17  -0.13  -0.09  -0.04  0.00 |
|                                                                          |
| 0.00  -0.06  -0.13  -0.20  -0.28  -0.32  -0.28  -0.20  -0.13  -0.06  0.00 |
|                                                                          |
| 0.00  -0.08  -0.17  -0.28  -0.41  -0.55  -0.41  -0.28  -0.17  -0.08  0.00 |
|                                                                          |
| 0.00  -0.09  -0.19  -0.32  -0.55  -1.05  -0.55  -0.32  -0.19  -0.09  0.00 |
|                                                                          |
| 0.00  -0.08  -0.17  -0.28  -0.41  -0.55  -0.41  -0.28  -0.17  -0.08  0.00 |
|                                                                          |
| 0.00  -0.06  -0.13  -0.20  -0.28  -0.32  -0.28  -0.20  -0.13  -0.06  0.00 |
|                                                                          |
| 0.00  -0.04  -0.09  -0.13  -0.17  -0.19  -0.17  -0.13  -0.09  -0.04  0.00 |
|                                                                          |
| 0.00  -0.02  -0.04  -0.06  -0.08  -0.09  -0.08  -0.06  -0.04  -0.02  0.00 |
|                                                                          |
| 0.00  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00 |
```

adi

Description:

This function solves an elliptic partial differential equation by the alternating direction implicit method. If u is the unknown solution then the discretized form of the equation for this method is:

$$Lu = g \quad where \quad L = L_x + L_y \quad with$$
$$(L_x u)_{j,l} = a_{j,l}u_{j-1,l} + b_{j,l}u_{j,l} + c_{j,l}u_{j+1,l}$$
$$(L_y u)_{j,l} = d_{j,l}u_{j,l-1} + e_{j,l}u_{j,l} + f_{j,l}u_{j,l+1}$$

adi(a, b, c, d, e, f, g, u, k, alpha, beta) $\longrightarrow$ u

Where:

a is a real square matrix dimensioned to the grid size,
b is a real square matrix dimensioned to the grid size,
c is a real square matrix dimensioned to the grid size,
d is a real square matrix dimensioned to the grid size,
e is a real square matrix dimensioned to the grid size,
f is a real square matrix dimensioned to the grid size,
g is a real square matrix dimensioned to the grid size representing the right hand side,
u is a real square matrix dimensioned to the grid sized representing on input an initial guess for the solution and on output the computed solution,
k is an integer such that the number of initial iterations is 2^k.
alpha is a real number representing an upperbound for the modulus of the eigenvalues of L_x.
beta is a real number representing an upper bound for the eigenvalue of L_y.

Pre-variables:

adi_eps [1.0d-5] a floating point number representing the relative accuracy,
adi_jj [50] an integer upper bound for the grid size,
adi_maxits [100] the total number of allowed iterations,
adi_kk [6] an integer setting the number of iterations for each fixed eigenvalue,
adi_nrr [32] an integer being 2^{kk-1}.

Example:

In this example the same elliptic problem as that used for sor is solved using adi.

```
-> a : constant_matrix(-1,11,11);
-> c : a; d : a; f : a;
-> b :constant_matrix(2,11,11);
-> e : b;
-> pi:float(%pi);
-> alpha : 2*(1 - cos(pi/11))?
0.08
-> beta : 2*(1 - cos(pi*10/11))?
3.92
-> u : zero(11,11);
-> g : insert_elt(2d0,6,6,zero(11,11));
-> k:2; adi_eps:0.01;
-> adi(a,b,c,d,e,f,g,u,k,alpha,beta)?
| 0.00  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00 |
|                                                                            |
| 0.00  -0.02  -0.04  -0.06  -0.08  -0.09  -0.08  -0.06  -0.04  -0.02  0.00 |
|                                                                            |
| 0.00  -0.04  -0.09  -0.13  -0.17  -0.19  -0.17  -0.13  -0.09  -0.04  0.00 |
|                                                                            |
| 0.00  -0.06  -0.13  -0.20  -0.28  -0.32  -0.28  -0.20  -0.13  -0.06  0.00 |
|                                                                            |
| 0.00  -0.08  -0.17  -0.28  -0.41  -0.55  -0.41  -0.28  -0.17  -0.08  0.00 |
|                                                                            |
| 0.00  -0.09  -0.19  -0.32  -0.55  -1.05  -0.55  -0.32  -0.19  -0.09  0.00 |
|                                                                            |
| 0.00  -0.08  -0.17  -0.28  -0.41  -0.55  -0.41  -0.28  -0.17  -0.08  0.00 |
|                                                                            |
| 0.00  -0.06  -0.13  -0.20  -0.28  -0.32  -0.28  -0.20  -0.13  -0.06  0.00 |
|                                                                            |
| 0.00  -0.04  -0.09  -0.13  -0.17  -0.19  -0.17  -0.13  -0.09  -0.04  0.00 |
|                                                                            |
| 0.00  -0.02  -0.04  -0.06  -0.08  -0.09  -0.08  -0.06  -0.04  -0.02  0.00 |
|                                                                            |
| 0.00  0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00   0.00 |
```