Unbiased flux calibration for Single-dish radio telescopes

Dealing with frequency-dependence

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Overview

- Introduction: simplest flux calibration
- Position switching
  - Classic method
  - Unbiased technique
- Frequency switching
- Comparison PSW vs. FSW
Fundamental equation

\[ P[\text{counts}] = G[\text{counts}/K] \cdot (T_{\text{sou}}[K] + T_{\text{sys}}[K]) \]
Fundamental equation

\[ P[\text{counts}] = G[\text{counts}/K] \cdot (T_{\text{sou}}[K] + T_{\text{sys}}[K]) \]

Calibrating a system means to determine G
Most simple method

- Use known source to infer $G$

Assuming, the receiver is linear: $G \neq f(T)$
Most simple method

- Use known source to infer $G$
- Problem: $G$ is not perfectly stable
- Solution:
  - Use a reference $→$ Noise diode ($T_{\text{cal}}$)

Assuming, the receiver is linear: $G \not\equiv f(T)$
Using a noise diode

We now have

\[ P = G \cdot (T_{\text{sou}} + T_{\text{sys}}) \]

\[ P_{\text{cal}} = G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}}) \]

It follows

\[ \frac{T_{\text{sys}} + T_{\text{sou}}}{T_{\text{cal}}} = \frac{P}{P_{\text{cal}} - P} \]
Using a noise diode

We now have

\[ P = G \cdot (T_{\text{sou}} + T_{\text{sys}}) \]

\[ P_{\text{cal}} = G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}}) \]

It follows

\[ \frac{T_{\text{sys}} + T_{\text{sou}}}{T_{\text{cal}}} = \frac{P}{P_{\text{cal}} - P} \]

This is really noisy, because \( P_{\text{cal}} - P \ll P \)
Using a noise diode

We now have

\[ P = G \cdot (T_{sou} + T_{sys}) \]

\[ P^{cal} = G \cdot (T_{sou} + T_{sys} + T_{cal}) \]

It follows

\[ \frac{T_{sys} + T_{sou}}{T_{cal}} = \frac{P}{P^{cal} - P} \]

Identify \( G \cdot T_{cal} = P^{cal} - P \) as a slowly changing quantity (or even constant)

\[ \overline{G \cdot T_{cal}} = \langle P^{cal} - P \rangle \]
Using a noise diode

We now have

\[ P = G \cdot (T_{\text{sou}} + T_{\text{sys}}) \]

\[ P^{\text{cal}} = G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}}) \]

It follows

\[ \frac{T_{\text{sys}} + T_{\text{sou}}}{T_{\text{cal}}} = \frac{P}{G \cdot T_{\text{cal}}} \]

We still need to use a calibration source to infer \( T_{\text{cal}} \)!
First conclusion

Flux calibration is about knowing where one may safely average (or filter, or model) a quantity.
Again we have

\[ P = G \cdot (T_{\text{so}} + T_{\text{sys}}) \]

\[ P_{\text{cal}} = G \cdot (T_{\text{so}} + T_{\text{sys}} + T_{\text{cal}}) \]

But now everything is a function of frequency
Spectroscopy

Again we have

\[ P = G \cdot (T_{\text{sou}} + T_{\text{sys}}) \]
\[ P_{\text{cal}} = G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}}) \]

Try again

\[ \frac{T_{\text{sys}} + T_{\text{sou}}}{T_{\text{cal}}} = \frac{P}{P_{\text{cal}} - P} \]
Again we have

\[ P = G \cdot (T_{\text{sou}} + T_{\text{sys}}) \]

\[ P_{\text{cal}} = G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}}) \]

Try again

\[ \frac{T_{\text{sys}} + T_{\text{sou}}}{T_{\text{cal}}} = \frac{P}{P_{\text{cal}} - P} \]
Spectroscopy

Again we have

\[ P = G \cdot (T_{\text{sou}} + T_{\text{sys}}) \]

\[ P_{\text{cal}} = G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}}) \]

Try again

\[ \frac{T_{\text{sys}} + T_{\text{sou}}}{T_{\text{cal}}} = \frac{P}{P_{\text{cal}} - P} \]

Average in time? (needs more data)

Average in frequency? (bad idea, G strongly freq-dependent)
Position switching

Observe ON and OFF-source

\[ P_{\text{on}} = G \cdot (T_{\text{sou}} + T_{\text{sys}}) \]
\[ P_{\text{cal}} = G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}}) \]
\[ P_{\text{off}} = G \cdot (T_{\text{sys}}) \]
\[ P_{\text{off}} = G \cdot (T_{\text{sys}} + T_{\text{cal}}) \]
Position switching

Observe ON and OFF-source

\[
P_{\text{on}} = G \cdot (T_{\text{sou}} + T_{\text{sys}})
\]

\[
P_{\text{cal}} = G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}})
\]

\[
P_{\text{off}} = G \cdot (T_{\text{sys}})
\]

\[
P_{\text{off}} = G \cdot (T_{\text{sys}} + T_{\text{cal}})
\]

It follows

\[
\frac{T_{\text{sou}}}{T_{\text{sys}}} = \frac{P_{\text{on}} - P_{\text{off}}}{P_{\text{off}}}
\]
Position switching

Observe ON and OFF-source

\[ P_{\text{on}} = G \cdot (T_{\text{sou}} + T_{\text{sys}}) \]
\[ P_{\text{cal}} = G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}}) \]
\[ P_{\text{off}} = G \cdot (T_{\text{sys}}) \]
\[ P_{\text{off}} = G \cdot (T_{\text{sys}} + T_{\text{cal}}) \]

It follows

\[ \frac{T_{\text{sou}}}{T_{\text{sys}}} = \frac{P_{\text{on}} - P_{\text{off}}}{P_{\text{off}}} \]

However, this is a function of \( T_{\text{sys}} \)

Need to compute it!
PSW – Inferring $T_{sys}$

Observe ON and OFF-source

\[
P_{\text{on}} = G \cdot (T_{\text{sou}} + T_{\text{sys}})
\]
\[
P_{\text{cal}} = G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}})
\]
\[
T_{\text{off}} = G \cdot (T_{\text{sys}})
\]
\[
P_{\text{off}} = G \cdot (T_{\text{sys}} + T_{\text{cal}})
\]

Compute

\[
\frac{T_{\text{sys}}}{T_{\text{cal}}} = \frac{P_{\text{off}}}{P_{\text{cal}}^{\text{off}} - P_{\text{off}}^{\text{cal}}}
\]
PSW – Inferring $T_{\text{sys}}$

Observe ON and OFF-source

\[
\begin{align*}
    P_{\text{on}} &= G \cdot (T_{\text{sou}} + T_{\text{sys}}) \\
    P_{\text{cal}} &= G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}}) \\
    P_{\text{off}} &= G \cdot (T_{\text{sys}}) \\
    P_{\text{off}}^\text{cal} &= G \cdot (T_{\text{sys}} + T_{\text{cal}})
\end{align*}
\]

Compute

\[
\frac{T_{\text{sys}}}{T_{\text{cal}}} = \frac{P_{\text{off}}}{P_{\text{off}}^\text{cal} - P_{\text{off}}}
\]

Again, the denominator is too small

Need some kind of averaging
PSW – Inferring $T_{sys}$ – Classic

Observe ON and OFF-source

\[
P_{on} = G \cdot (T_{sou} + T_{sys})
\]
\[
P_{cal} = G \cdot (T_{sou} + T_{sys} + T_{cal})
\]
\[
P_{off} = G \cdot (T_{sys})
\]
\[
P_{cal} = G \cdot (T_{sys} + T_{cal})
\]

Compute

\[
\frac{T_{sys}}{T_{cal}} = \left\langle \frac{P_{off}}{P_{cal} - P_{off}} \right\rangle_{\nu}
\]
PSW – Inferring $T_{\text{sys}}$ – Classic

Observe ON and OFF-source

\[
P_{\text{on}} = G \cdot (T_{\text{sou}} + T_{\text{sys}})
\]
\[
P_{\text{on}}^{\text{cal}} = G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}})
\]
\[
P_{\text{off}} = G \cdot (T_{\text{sys}})
\]
\[
P_{\text{off}}^{\text{cal}} = G \cdot (T_{\text{sys}} + T_{\text{cal}})
\]

Compute

\[
\frac{T_{\text{sys}}}{T_{\text{cal}}} = \left\langle \frac{P_{\text{off}}}{P_{\text{off}}^{\text{cal}} - P_{\text{off}}} \right\rangle
\]
\[
\frac{T_{\text{sys}}}{T_{\text{cal}}} = \left\langle \frac{P_{\text{off}}^{\text{cal}} - P_{\text{off}}}{P_{\text{off}}} \right\rangle^{-1}
\]
PSW – Classic solution

Observe ON and OFF-source

\[ P_{on} = G \cdot (T_{sou} + T_{sys}) \]
\[ P_{cal} = G \cdot (T_{sou} + T_{sys} + T_{cal}) \]
\[ P_{off} = G \cdot (T_{sys}) \]
\[ P_{cal} = G \cdot (T_{sys} + T_{cal}) \]

"Classic" solution

\[ \frac{T_{sou}}{T_{cal}} = \frac{T_{sys}}{T_{cal}} \frac{P_{on} - P_{off}}{P_{off}} \]
Observe ON and OFF-source

\[
\begin{align*}
    P_{\text{on}} &= G \cdot (T_{\text{sou}} + T_{\text{sys}}) \\
    P_{\text{off}} &= G \cdot (T_{\text{sys}}) \\
    P_{\text{cal}} &= G \cdot (T_{\text{sou}} + T_{\text{cal}}) \\
    P_{\text{cal}} &= G \cdot (T_{\text{sys}} + T_{\text{cal}})
\end{align*}
\]

“Classic” solution

\[
\frac{T_{\text{sou}}}{T_{\text{cal}}} = \frac{\overline{T_{\text{sys}}} P_{\text{on}} - P_{\text{off}}}{T_{\text{cal}} P_{\text{off}}}
\]

But: \( T_{\text{sys}} \) and \( T_{\text{cal}} \) are frequency-dependent!
Typical $T_{\text{sys}}$ and $T_{\text{cal}}$ curves
Typical $T_{\text{sys}}$ and $T_{\text{cal}}$ curves

P13mm-XFFTS: $T_{\text{cal}}$ using NGC7027

Frequency [MHz]

$T_{\text{cal}}$ [K]

18000 19000 20000 21000 22000 23000 24000 25000 26000
Second conclusion

We must account for the frequency dependence, if

- Bandwidth is large
- $T_{\text{sys}}$ or $T_{\text{cal}}$ change rapidly
- Relative calibration is important (i.e., line ratios)
PSW – Unbiased method

Observe ON and OFF-source

\[
\begin{align*}
P_{\text{on}} &= G \cdot (T_{\text{sou}} + T_{\text{sys}}) \\
P_{\text{off}} &= G \cdot (T_{\text{sys}}) \\
P_{\text{on}}^\text{cal} &= G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}}) \\
P_{\text{off}}^\text{cal} &= G \cdot (T_{\text{sys}} + T_{\text{cal}})
\end{align*}
\]

It follows

\[
\frac{T_{\text{sou}}}{T_{\text{sys}}} = \frac{P_{\text{on}} - P_{\text{off}}}{P_{\text{off}}}
\]

Start with the same base equation
PSW – Unbiased method

Observe ON and OFF-source

\[ P_{\text{on}} = G \cdot (T_{\text{sou}} + T_{\text{sys}}) \]
\[ P_{\text{cal}} = G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}}) \]
\[ P_{\text{off}} = G \cdot (T_{\text{sys}}) \]
\[ P_{\text{off}} = G \cdot (T_{\text{sys}} + T_{\text{cal}}) \]

It follows

\[ \frac{T_{\text{sys}}}{T_{\text{cal}}} = \frac{P_{\text{off}}}{P_{\text{cal}} - P_{\text{off}}} \]

Just use another method to calculate Tsys/Tcal
PSW – Unbiased method

Observe ON and OFF-source

\[ P_{\text{on}} = G \cdot (T_{\text{sou}} + T_{\text{sys}}) \]
\[ P_{\text{on}}^\text{cal} = G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}}) \]
\[ P_{\text{off}} = G \cdot (T_{\text{sys}}) \]
\[ P_{\text{off}}^\text{cal} = G \cdot (T_{\text{sys}} + T_{\text{cal}}) \]

It follows

\[ \frac{T_{\text{sys}}}{T_{\text{cal}}} = \left[ \frac{P_{\text{off}}^\text{cal} - P_{\text{off}}}{P_{\text{off}}} \right]^{-1} \]

Model this quantity and invert afterwards.
Observe ON and OFF-source

\[
\begin{align*}
    P_{\text{on}} &= G \cdot (T_{\text{sou}} + T_{\text{sys}}) \\
    P_{\text{cal}} &= G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}}) \\
    P_{\text{off}} &= G \cdot (T_{\text{sys}}) \\
    P_{\text{off}}^\text{cal} &= G \cdot (T_{\text{sys}} + T_{\text{cal}})
\end{align*}
\]

It follows

\[
\frac{T_{\text{sys}}}{T_{\text{cal}}} = \left[ \frac{P_{\text{cal}}^\text{off} - P_{\text{cal}}^\text{off}}{P_{\text{off}}^\text{off}} \right]^{-1}
\]

Model this quantity and invert afterwards
PSW – Unbiased method

Possible methods to model $T_{\text{cal}}/T_{\text{sys}}$

- Simple binning
- Filtering (e.g., Wiener or Gauss)
- Curve fitting (e.g., polynomials)
PSW – Unbiased method – Results

Observe ON and OFF-source

\[ P_{\text{on}} = G \cdot (T_{\text{sou}} + T_{\text{sys}}) \]
\[ P_{\text{cal}} = G \cdot (T_{\text{sou}} + T_{\text{sys}} + T_{\text{cal}}) \]
\[ P_{\text{off}} = G \cdot (T_{\text{sys}}) \]
\[ P_{\text{calc}} = G \cdot (T_{\text{sys}} + T_{\text{cal}}) \]

Using

\[ \frac{T_{\text{sou}}}{T_{\text{sys}}} = \frac{P_{\text{on}} - P_{\text{off}}}{P_{\text{off}}} \]

With

\[ \frac{T_{\text{sys}}}{T_{\text{cal}}} = \left[ \frac{P_{\text{calc}} - P_{\text{off}}}{P_{\text{off}}} \right]^{-1} \]
PSW – Unbiased method – Results

Why does the correct result look so much worse?

Because we didn't account for $T_{\text{cal}}$ yet!
PSW – Unbiased method – Recipe

How to calibrate your spectral data

- Position switch on (bright) continuum calibrator
  - Reduce using the advanced method to infer $T_{\text{sou}}/T_{\text{cal}}$
  - We know $S_{\text{calib}} \rightarrow$ expect $T_A(S_{\text{calib}}) = T_{\text{sou}}$ (A. Kraus talk)
  - Calculate $T_{\text{cal}}$

- Position switch on target source
  - Reduce using the advanced method to infer $T_{\text{sou}}/T_{\text{cal}}$
  - Apply $T_{\text{cal}}$ spectrum
PSW – Unbiased method – Results

Scan: 1733 Subscan: 1

Tnew
Tclassic
PSW – Unbiased method – Results

Correct continuum slope!
PSW – Unbiased method – Results

RRL: H109α
PSW - Unbiased method - Results

RRL: H109α

Scan: 1733 Subscan: 1

Tnew = 3.51
Tclassic = 3.71
PSW – Unbiased method – Results

RRL: H112α

Classic: Line ratio wrong by 10% (systematically!)
Third conclusion

Extremely simple to incorporate freq. dependence

But

- Modeling not always robust, may need supervision (e.g., in case of standing waves)
- $T_{sys}$ may not be stable between ON and OFF
- Weather can hurt a lot!
  → Solution: cross-scanning
- Frequency dependence also for opacity, Elevation-gain curve, taper function
- Must ensure receiver linearity (especially for $T_{cal}$ determination)
Receiver linearity

\[ G \cdot T_{\text{cal}} = \langle P^{\text{cal}} - P \rangle \] should not depend on \( T_{\text{sol}} \).
Frequency switching

Use a shift in frequency to obtain a reference spectrum. This can only remove the IF part of the gain!

\[
P_{\text{sig}}(\nu) = G_{\text{IF}} G_{\text{RF},-} \left[ T_{\text{sys},-} + T_{\text{cal}} \right]
\]

\[
P_{\text{ref}}^{[\text{cal}]}(\nu) = G_{\text{IF}} G_{\text{RF},+} \left[ T_{\text{sys},+} + T_{\text{cal}} \right]
\]
Frequency switching

Equations get lengthy...

\[ T_{\text{sou},-} - T_{\text{sou},+} + \Delta T_{\text{sys}, \pm} = \left( T_{\text{sou},+} + T_{\text{sys},+} \right) \frac{P_{\text{sig}}^{[\text{cal}]} - P_{\text{ref}}^{[\text{cal}]}}{P_{\text{ref}}^{[\text{cal}]}} \equiv \tilde{T}_{\text{sig}}^{[\text{cal}]} \]

\[ T_{\text{sou},+} - T_{\text{sou},-} - \Delta T_{\text{sys}, \pm} = \left( T_{\text{sou},-} + T_{\text{sys},-} \right) \frac{P_{\text{ref}}^{[\text{cal}]} - P_{\text{sig}}^{[\text{cal}]} }{P_{\text{sig}}^{[\text{cal}]}} \equiv \tilde{T}_{\text{ref}}^{[\text{cal}]} \]

\[ \frac{1}{2} \left[ \tilde{T}_{\text{sig}}^{[\text{cal}]} (\nu + \Delta \nu) + \tilde{T}_{\text{ref}}^{[\text{cal}]} (\nu - \Delta \nu) \right] = \]

\[ T_{\text{sou}}(\nu) - \frac{1}{2} T_{\text{sou}}(\nu + 2 \Delta \nu) - \frac{1}{2} T_{\text{sou}}(\nu - 2 \Delta \nu) + T_{\text{sys}}^{[\text{cal}]}(\nu) - \frac{1}{2} T_{\text{sys}}^{[\text{cal}]}(\nu + 2 \Delta \nu) - \frac{1}{2} T_{\text{sys}}^{[\text{cal}]}(\nu - 2 \Delta \nu) \]
Frequency switching

But what about calibration?
Frequency switching

But what about calibration?

Short answer:

It's “possible”, but don't!
Frequency switching

Problems/drawbacks of frequency-dependent calibration

- Modeling rarely robust (also needs flagging of spectral lines)
- Source continuum not reconstructed
- Only viable if $G_{RF,+} = G_{RF,-} \leftarrow$ never true in reality!
- Produces awful baselines (yet correct)

See Winkel, Kraus & Bach, A&A 540, 2012 for details
PSW vs. FSW


Simulated inputs
PSW vs. FSW

Unbiased PSW

PSW vs. FSW


Classic PSW
PSW vs. FSW


Unbiased FSW

$T$ [K]

$\nu$ [MHz]
Summary

- Frequency-dependence must not be neglected
- Easy to incorporate for PSW, but not for FSW
- Can be less robust than classic scheme → Supervision may be necessary
- Calibration needs time
A final warning

Do not use the Gildas/class fold command!

See Winkel, Kraus & Bach, A&A 540, 2012 for details
A final warning

Do not use the Gildas/class fold command!

See Winkel, Kraus & Bach, A&A 540, 2012 for details