Evolution of the $M_\bullet - \sigma$ relation

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May 14-18, 2018, IAU 342 symposium, Noto, Italy

1. Abstract

Black holes at the centers of the galaxies grow by mainly two processes: accretion of gas and consumption of stars. In the case of gas accretion with cooling sources, the flow is momentum driven, after which the black hole reaches a saturated mass and subsequently, it grows only by consumption of stars. We have studied the evolution of the black hole mass and spin with the initial seed mass as a function of redshift in a ΛCDM cosmology. For the stellar ingestion, we have assumed a power-law density profile for the stellar cusp in the frame work of relativistic loss cone theory that includes the effect of the black hole spin. We predict the impact of the evolution on the $M_\bullet - \sigma$ relation and compare it with available observations.

2. Consumption of stars in power law cases

- The distribution function of stars in a galaxy where mass density follows single power law profile is given by,

$$f(E) = \frac{3-\gamma}{8}\sqrt{\frac{2}{\pi^5}}\frac{\Gamma(\gamma+1)}{\Gamma(\gamma-\frac{1}{2})}\frac{M_\bullet}{m_\star}\frac{\phi_0^{\frac{3}{2}}}{(GM_\bullet)^3}\left(\frac{|E|}{\phi_0}\right)^{\gamma-\frac{3}{2}},$$

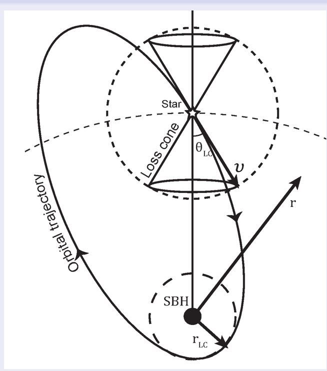
  where, $\phi_0 = \frac{GM_\bullet}{r_m}$.
- Number of stars at energies E to $E + \mathrm{d}E$ and angular momentum L to $L + \mathrm{d}L$ is

$$N(E, L)\mathrm{d}E\mathrm{d}L = 8\pi^2 L f(E, L)P(E, L)\mathrm{d}E\mathrm{d}L,$$

  where, $P(E, L)$ is the radial orbital period.
- Therefore number of stars within loss cone will be

$$N_{lc}(E)\mathrm{d}E = 4\pi^2 L_{lc}^2(E)f(E)P(E)\mathrm{d}E.$$

3. Rate of consumption from loss cone theory



- The rate of consumption of stars within the loss cone is

$$F_{flc}(E) = 4\pi^2 L_{lc}^2(E)f(E).$$

- Integration of this over all energies gives the total rate of consumption in the loss cone,

$$\dot{N_{flc}} = \int_{-\infty}^{\phi_0} F_{flc}(E)dE.$$

$$m_\star \dot{N_{flc}} = \frac{3-\gamma}{8}\sqrt{\frac{1}{2\pi}\frac{\Gamma(\gamma+1)}{\Gamma(\gamma-\frac{1}{2})}}\frac{L_{lc}^2}{r_m}\frac{1}{r_m}\left(\frac{GM_\bullet}{r_m^3}\right)^{\frac{1}{2}} M_\bullet$$

4. Calculation of tidal radius

- Tidal radius calculation (BM18 in preparation):

$$\left.\frac{\partial^2 V_{eff}}{\partial r^2}\right|_{r=r_t} = \frac{4\pi}{3}\eta^2 G\rho,$$

  where, η is the form factor of order unity, depends on the type of star.
- In natural units this equation finally leads to

$$\left.-\frac{1}{x^3} + \frac{3\tilde{l}^2}{x^4} - 6\frac{(\frac{a}{2} - \tilde{l})^2}{x^5}\right|_{x=x_t} = \eta^2\tilde{\rho}.$$

- Writing $\tilde{\rho}$ as approximately M_8^2 (assuming the star to be of solar type), we solve the equation for x_t as a function of a and M_8, we have used $\eta = 0.844$ as done by Merritt(2013).
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$$x_H = \frac{1}{2}(1 + \sqrt{1-a^2}).$$

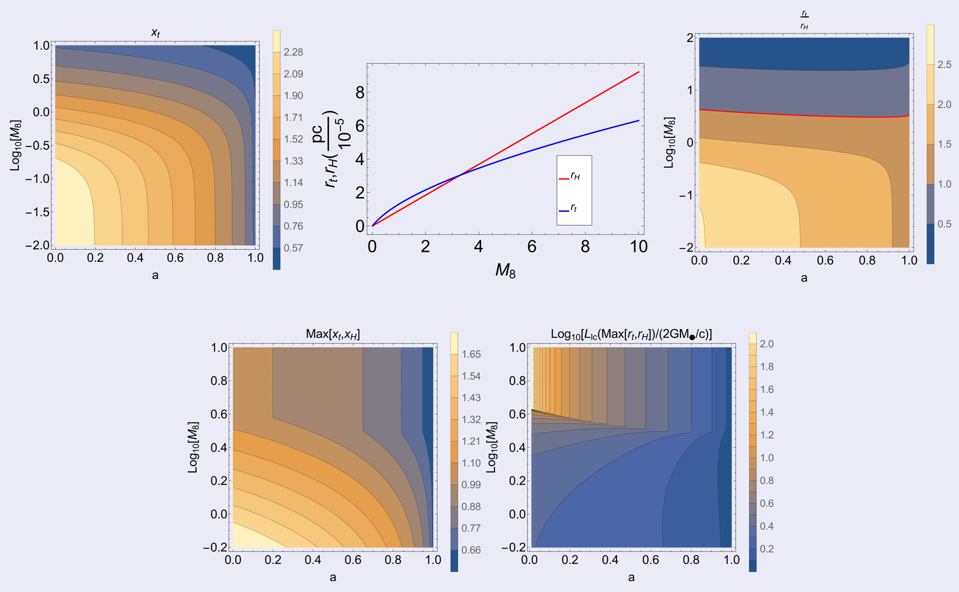

Figure : Tidal radius and the horizon (in units of R_s) as a function of (a, M_8) and a, their ratio as function of a, M_8 (up), x_{lc} and L_{lc} (in units of $\frac{2GM_\bullet}{c}$)at x_{lc} (down).

5. Steady loss cone theory

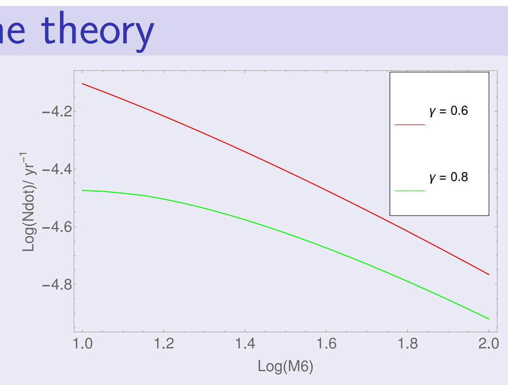

Figure : The variation of $\dot{N}$ with M_6 for two different values of γ with es_{min} = -100 and σ = 200 km/sec.

- From Mageshwaran & Mangalam (2015) for the steady loss cone

$$\frac{\mathrm{d}^2\dot{N}_t}{\mathrm{d}\bar{e}\mathrm{d}l^2\mathrm{d}m} = 4\pi^2 s_t^{-1}\sigma^2\epsilon(m)f_*(\bar{e}, M_\bullet, m)L_{lc}^2(\bar{e})F(\chi = 1, l),$$

  where, $s_t = r_t/r_h$, $\bar{e} = E/(GM_\bullet/r_t)$.
- We finally arrive at

$$\frac{\mathrm{d}\dot{N}}{\mathrm{d}\epsilon} = \frac{8\pi r_t^2\sigma^7}{G^3 M_\bullet^2 < m_* >}\left(\frac{r_h}{r_t} - \epsilon\right)g(\epsilon)\frac{f(\epsilon)}{1 + q^{-1}\zeta(q)\log(1/R_{lc})}.$$

6. Evolution of the black hole (BM18 in preparation)

- The spin evolution equation of black hole is given by (Mangalam., 2015):

$$\frac{\mathrm{d}j}{\mathrm{d}t} = \frac{\dot{M_0}}{M_\bullet}\Big(\ell_I(j) - 2\epsilon(j)j\Big) + r^3(j)\frac{\mathcal{G}_0}{\mathcal{J}_0}.$$

  The last term is due to BZ torque, where, $r(j) = 1 + \sqrt{1-j^2}$, Torque,

$$\mathcal{G}_0 = \frac{m^3}{8}B_\perp^2 f = 4\times 10^{46} f B_4 M_8^3(\mathrm{erg}),$$

  Angular momentum budget,

$$\mathcal{J}_0 = cM_\bullet mj = 9\times 10^{64} M_8^2(\mathrm{g\ cm^2\ s^{-1}}).$$

- Mass evolution equation is :

$$\frac{\mathrm{d}M_\bullet}{\mathrm{d}t} = \epsilon(j)\dot{M_0},$$

$$\epsilon(j) = \begin{cases} 1 - \epsilon_I(j) & \text{for } M_\bullet < M_c \\ 0 & \text{for } M_\bullet \geq M_c, \end{cases}$$

  where, ϵ is the mass accretion efficiency and it is dependent on the spin parameter j defined as,

$$\epsilon(j) = \begin{cases} \epsilon_I(j) & \text{for } M_\bullet < M_c \\ 1 & \text{for } M_\bullet \geq M_c, \end{cases}$$

  where,

$$\epsilon_I(j) = \frac{z^2(j) - 2z(j) + j\sqrt{z(j)}}{z(j)(z^2(j) - 3z(j) + 2j\sqrt{z(j)})^{1/2}},$$

  where,

$$z = \frac{r_{ms}}{M_\bullet} = z(j) = 3 + Z_2 - \sqrt{(3-Z_1)(3+Z_1+2Z_2)},$$

  with $Z_1 = 1 + (1-j^2)^{1/3}((1+j)^{1/3} + (1-j)^{1/3})$ and $Z_2 = (3j^2 + Z_1^2)^{1/2}$.
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$$l_I = \begin{cases} \frac{z^2(j) - 2j\sqrt{z(j)} + j^2}{z^{1/2}(j)(z^2(j) + 2j\sqrt{z(j)} - 3z(j))^{1/2}} & \text{for } M_\bullet < M_c \\ L_{lc}^{rel} & \text{for } M_\bullet \geq M_c, \end{cases}$$

$$\dot{M_0} = \dot{M_g} + \dot{M_*} + \dot{M_m} = \dot{M_g} + m_\star\dot{N}_{flc} + \dot{M_m},$$

where,

$$\dot{M_g} = k_1 M_\bullet, \text{where, } \mathrm{k_1} = \frac{\eta 4\pi \mathrm{Gm_p}}{\sigma_\mathrm{e}\mathrm{c}}.$$

For full loss cone,

$$L_{lc} = \begin{cases} \sqrt{2GM_\bullet r_{lc}} & \text{without spin} \\ \frac{2GM_\bullet}{c}\left(\frac{-j\pm\sqrt{j^2+4(x-1)(x^2+\frac{j^2}{4})}}{2(x-1)}\right) & \text{with spin} \end{cases}$$

where j is the spin parameter of the black hole, r_{lc} is the losscone radius and x is the radius of the loss cone in Schwarzchild radius unit which is taken as Max$[r_t, r_H]$.

$$\frac{\mathrm{d}M_m}{\mathrm{d}t} = A_t M\int_q^1 F(q)qdq,$$

where, $q = m/M$, $A_t(z, M) = 0.02\mathrm{Gyr}^{-1}(1+z)^{2.2}M_{12}^b$ with $b = 0.15$ and M_{12} = Mass in units of $10^{12}h^{-1}M_\odot$ with $h = 0.7$, $F(q) = q^{-c}(1-q)^d$ with $c = 0.5$ and $d = 1.3$.
From Gammie (2004) due to the effect of minor mergers spin evolution of the black hole

$$\frac{\mathrm{d}j}{\mathrm{d}M_\bullet} = \frac{j}{M_\bullet}\left(-\frac{7}{3} + \frac{9q}{\sqrt{2}j^2}\right).$$

$$t(z) = \frac{1}{H_0}\int_{1/(1+z_f)}^{1/(1+z)} da\frac{1}{\sqrt{\Omega_m a^{-1} + \Omega_\Lambda a^2}},$$

where, z_f is the formation redshift and H_0 is the present day Hubble constant.

$$t(z) = t_z(z) - t_z(z_f),$$

where,

$$t_z(z) = \frac{1}{H_0}\frac{2}{3}\frac{1}{\sqrt{1-\Omega_m}}log\left[\sqrt{1-\Omega_m}\sqrt{\Omega_m - \frac{\Omega_m - 1}{(1+z)^3}} - (\Omega_m - 1)\left(\frac{1}{1+z}\right)^{\frac{3}{2}}\right].$$

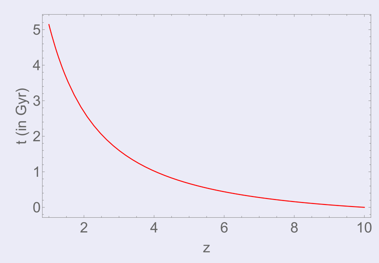
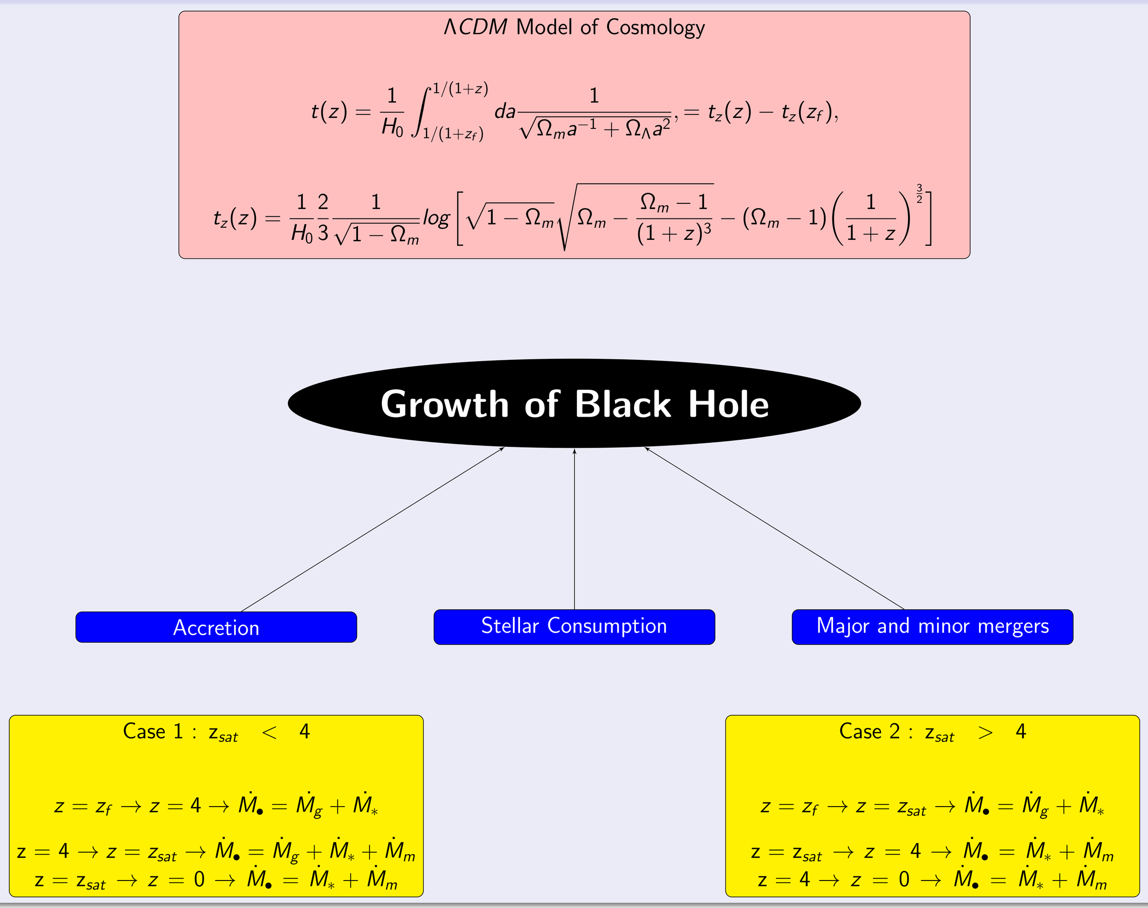


Figure : Age of universe is plotted in Gyr as a function of redshift in ΛCDM cosmology.

7. Flowchart



8. Impact on the mass and spin evolution

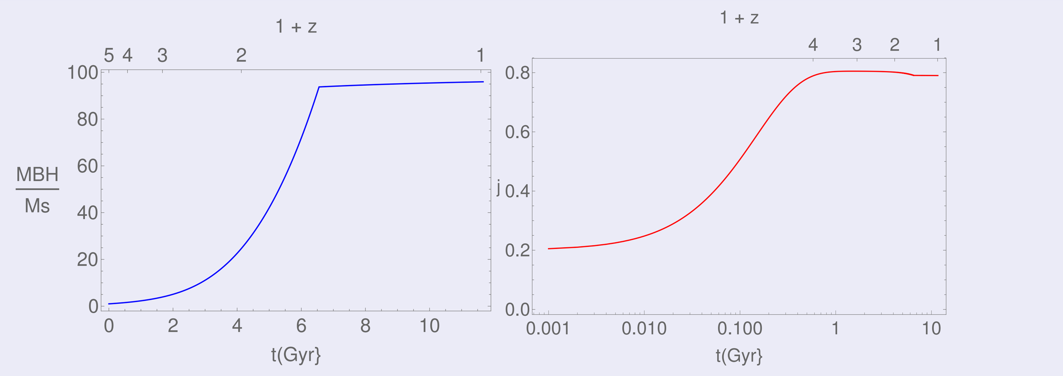


Figure : $M_\bullet$ and$j$ vs t (gyr) plot for $\eta = 0.1$ and $B_4 = 4$ when there is accretion and stellar consumption, merger and BZ torque present with $q = 0.1$, $j_s = 0.2$, $z_f = 4$ and $\gamma = 1.5$ with no prior assumption of $M_\bullet - \sigma$ relation.

9. Data used

Galaxy	$M_\bullet$ in $M_\odot$ (in pc)	σ (km/sec)
NGC 3379	1.36×10^8	163
NGC 3377	2.60×10^7	148
NGC 4486	1.88×10^9	305
NGC 4551	3.77×10^7	153
NGC 4472	1.17×10^9	382
NGC 3115	1.70×10^8	172
NGC 4467	4.93×10^6	77
NGC 4365	6.77×10^8	371
NGC 4636	5.80×10^8	251
NGC 4889	2.99×10^9	331
NGC 4464	1.12×10^7	112

This galaxies are selected from Wang & Merritt (2004). We took the data for Nuker intensity profiles of these galaxies and computed the LOS velocity dispersion from distribution function of stars considering $M_\bullet$ as a fraction of M_{Bulge}.

10. Impact on the $M_\bullet - \sigma$ relation

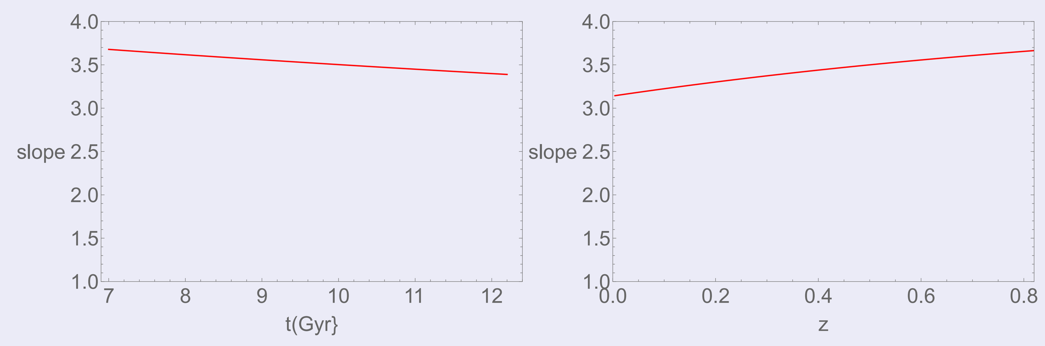


Figure : Evolution of the index of the $M_7 - \sigma_{100}$ relation with time and redshift for $\gamma = 1.5$, $M_s = 10^5 M_\odot$, $j_i = 0.2$, $B_4 = 5$, $z_f = 6$ from saturation time till present for steady loss cone assuming $M_\bullet - \sigma$ relation.

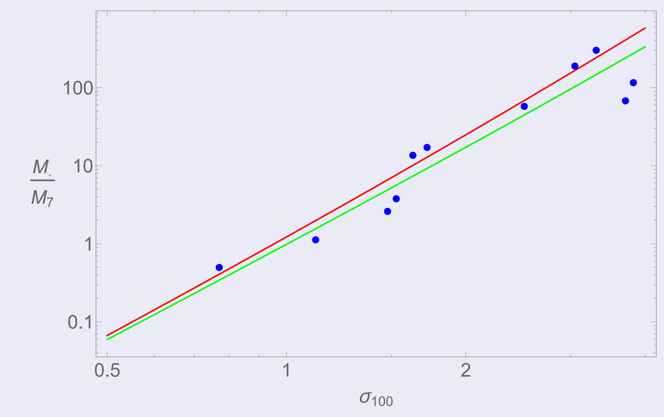


Figure : Plot of $\frac{M_\bullet}{M_7}$ vs σ_{100} for two different redshifts ($z = 0.002$, red and $z = 1$, green). We have overplotted the data obtained from our calculation (Bhattacharyya & Mangalam, 2018) for those 12 elliptical galaxies (their redshift lies in the range 0.004 - 0.002).

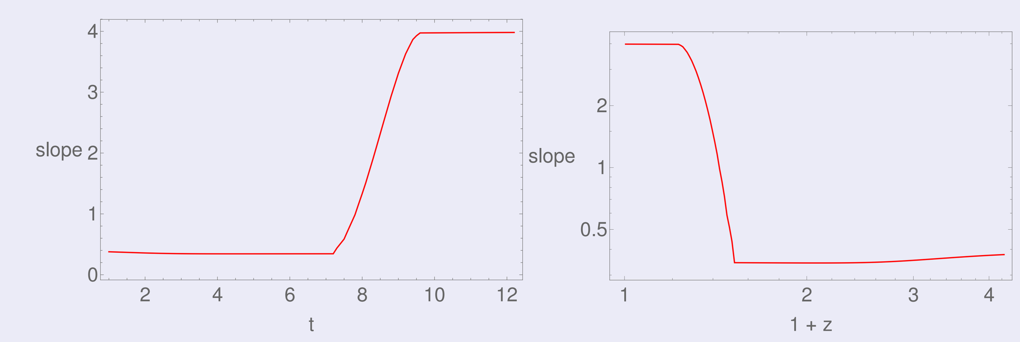


Figure : Evolution of the index of the $M_7 - \sigma_{100}$ relation with time and redshift for $\gamma = 1.5$, $M_s = 10^5 M_\odot$, $j_i = 0.2$, $B_4 = 5$, $z_f = 6$ with no prior assumption of $M_\bullet - \sigma$ relation.

11. Summary and Conclusions

1. We have studied the relativistic as well as the non relativistic evolution of black hole mass and spin.
2. The inputs to the growth of mass and spin evolution of black hole are accretion, stellar ingestion, major and minor mergers.
3. For the calculation of star consumption rate of the black hole, we assumed that the galaxy cusp follows a single power law density profile for the cases with and without spin for both full and steady loss cone theory. For practical purposes, the steady loss cone model is more appropriate.
4. We have determined the critical mass of black hole as a function of spin. Its value is $\simeq 3\times 10^8\ M_\odot$.
5. We have considered the merger to be effective from $z \leq 4$. The peak of merger activity is around $z = 4$ and before that the merger activity is negligible.
6. We have obtained the evolution of black hole as a function of redshift using cosmological ΛCDM model for different values of the spin parameter, seed masses and different formation redshifts.
7. We have compared our model with the available observations of z, $M_\bullet$ and $\sigma_{||}$ of 11 elliptical galaxies which follow $M_\bullet - \sigma$ relation and we are able to explain the observations from our model.
8. We have computed the evolution of the $M_\bullet - \sigma$ relation with redshift by deriving the evolution of the slope and intercept of log(M_7) vs log(σ_{100}) plot.
9. Using formula given by Shankar (2009) for renormalization, we show the $M_\bullet - \sigma$ relation to hold with $\alpha \simeq$ 0.24 - 0.34 till $z \simeq 1 - 1.2$ for the index (p) lying between 4 - 5.
10. Data from surveys at high redshift for example from TMT can be used to probe the $M_\bullet - \sigma$ relation at high redshift.

References

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