Sensitivity of LOFAR antenna elements

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Background

- System equivalent flux (SEFD) is a well know fundamental performance measure for antennas
- *But* SEFD theory for dual-polarized antennas measurement of Stokes I is still lacking...
  - Usually one just writes (Wrobel1999)

$$SEFD(dualPol)=\frac{SEFD(ant_1)}{\sqrt{2}}$$

  - This assumes both ant. gains are equal & no pol-leakage
  - However in practice no dual-pol ant. have equal gains and furthermore there is cross-pol leakage
    - Real dual-pol characterised by finite intrinsic cross-pol ratio (IXR)
  - In praticular Lofar has variable IXR with elevation

Goal

- Develope a formula for
  - the sensitivity for realistic (finite IXR) dual-polarized antennas
  - & the polarized sensitivity
- In particular study the elevation dependence of the sensitivity of Lofar
- Since gain does not vary with elevation for nominal beamforming only element beam is important for direction dependence

Radio interferometric measurement equation (RIME)

- RIME is based on 2x2 complex Jones formalism (Hamaker1996)
- It is a matrix relation between brightness and visibility
- Here we assume on-axis imaging
- Imaging and calibration is just matrix inversion of the RIME

$$\boldsymbol{\Phi} = \boldsymbol{J}\,\boldsymbol{B}\,\boldsymbol{J}^{H} + \boldsymbol{N}$$

$$\hat{\boldsymbol{B}} = \boldsymbol{J}^{-1}\boldsymbol{\Phi}\,\boldsymbol{J}^{-H}$$

Polarimetric definition of System Equivalent Flux Density

- Normal assumption for SEFD is that there is no source in beam
- Image RMS is due then due to calibrated noise
- For the Stokes I flux, this is the variance of the trace of the calibrated noise matrix

$$\boldsymbol{B}=\boldsymbol{0} \Rightarrow \quad \Delta \boldsymbol{B}=\boldsymbol{J}^{-1} \boldsymbol{N} \boldsymbol{J}^{-H}$$

$$SEFD = Var\left(Tr\left(\Delta \boldsymbol{B}\right)\right)$$

Ansats to a useful generic polarimeter model

- Any Jones matrix can be singular value decomposed as a matrix with scalar amplitude gain and one cross-polarization ratio

$$\boldsymbol{J} = \boldsymbol{U}\,\boldsymbol{J}'\,\boldsymbol{V}^{H}$$

- Two unitary transformations represent unique antenna and sky coordinates resp.

$$\boldsymbol{J}' = g\begin{pmatrix} 1 & d \\ d & 1 \end{pmatrix}$$

- Assume that noise in is independent in both antennas and equal

$$\boldsymbol{N} = N\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The symmetric 2 parameter Jones model

- Besides the 2 coordinate transforms there are only 2 real, positive parameters:

$$J' = g\begin{pmatrix} 1 & d \\ d & 1 \end{pmatrix}$$

  - d is directly related to polarization leakage and the cross-polarization ratio is the ”intrinsic cross-polarization ratio” IXR (Carozzi &Woan IEEE TAP 2011)

$$IXR = \frac{1}{d^2}$$

  - g is directly related to the total gain G

$$G = g^2\left(1 + d^2\right)$$

- Thus from 8 real parameters we have now 2 parameters relevant to polarimetric performance

SEFD for our polarimeter model

- Plugging the symetric Jones form into the SEFD formula we obtain

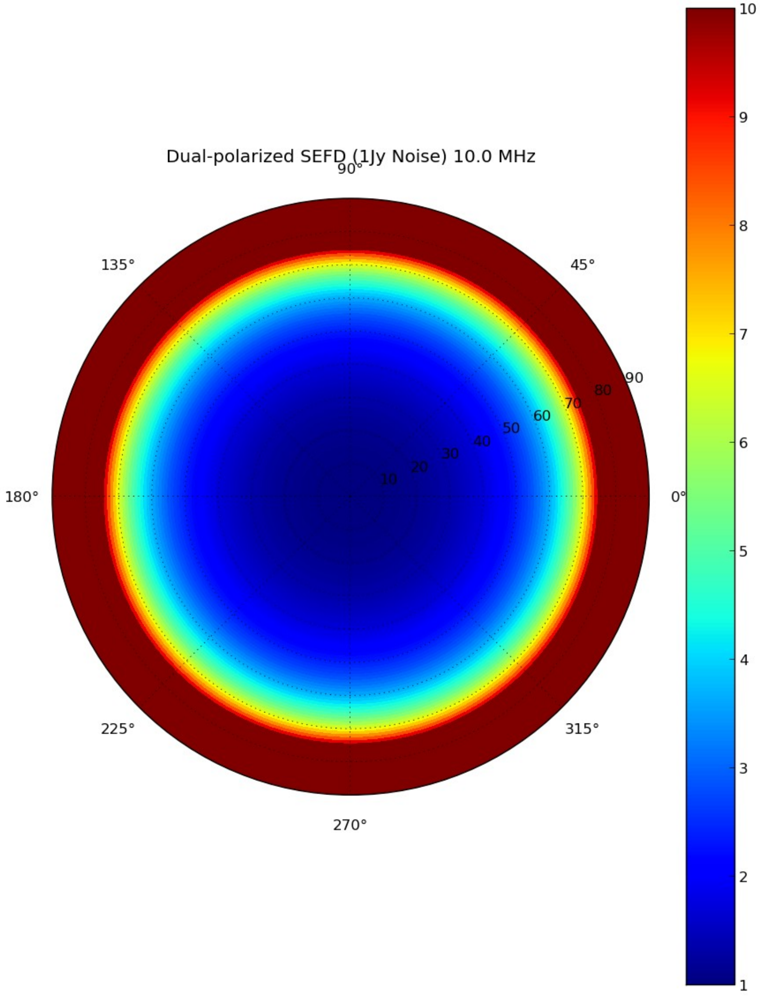
$$SEFD(\hat{I}) = \frac{N}{\sqrt{2}\,G}\left(\frac{(IXR+1)\sqrt{IXR^2+6\,IXR+1}}{(IXR-1)^2}\right)$$

- Scalar SEFD is factor on the left
- Factor on the right is new and account for finite IXR
  - for infinite IXR it is 1

Hamaker Model SEFD

- The reference antenna gain model for Lofar is known as the *Hamaker model*
- It is based on a EM simulation software
- With a monomial expansion in zenith angle (i.e. Taylor exp) and a Fourier series expansion in azimuth
- And a polynomial fit in frequency

10 MHz


Dual-polarized SEFD (1Jy Noise) 10.0 MHz
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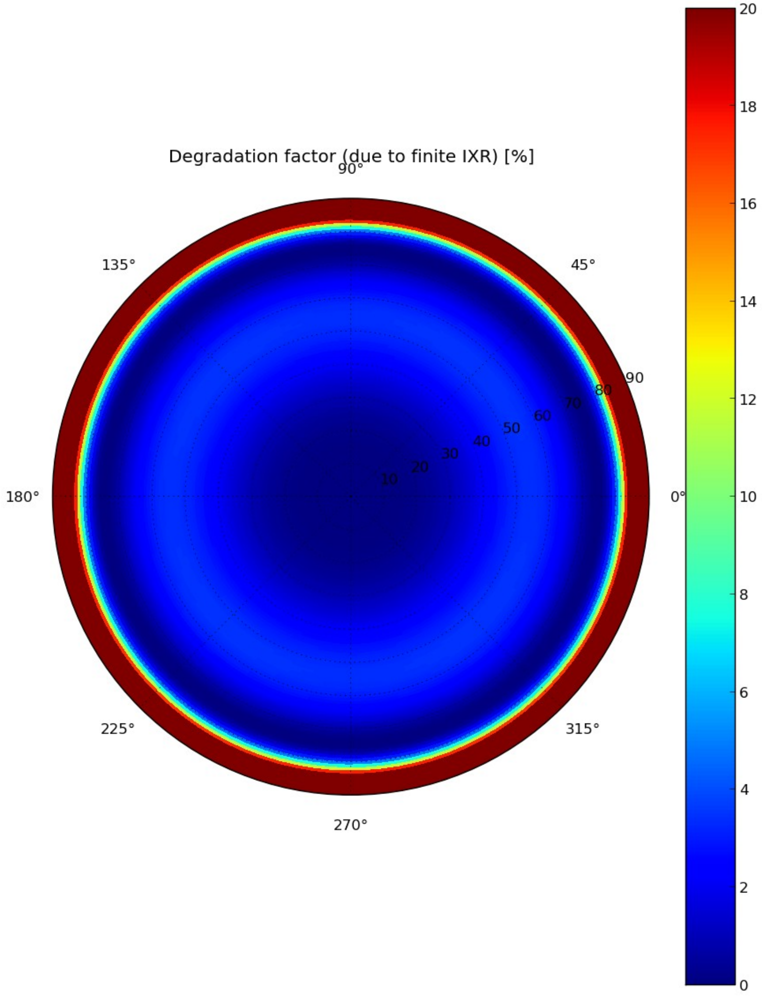
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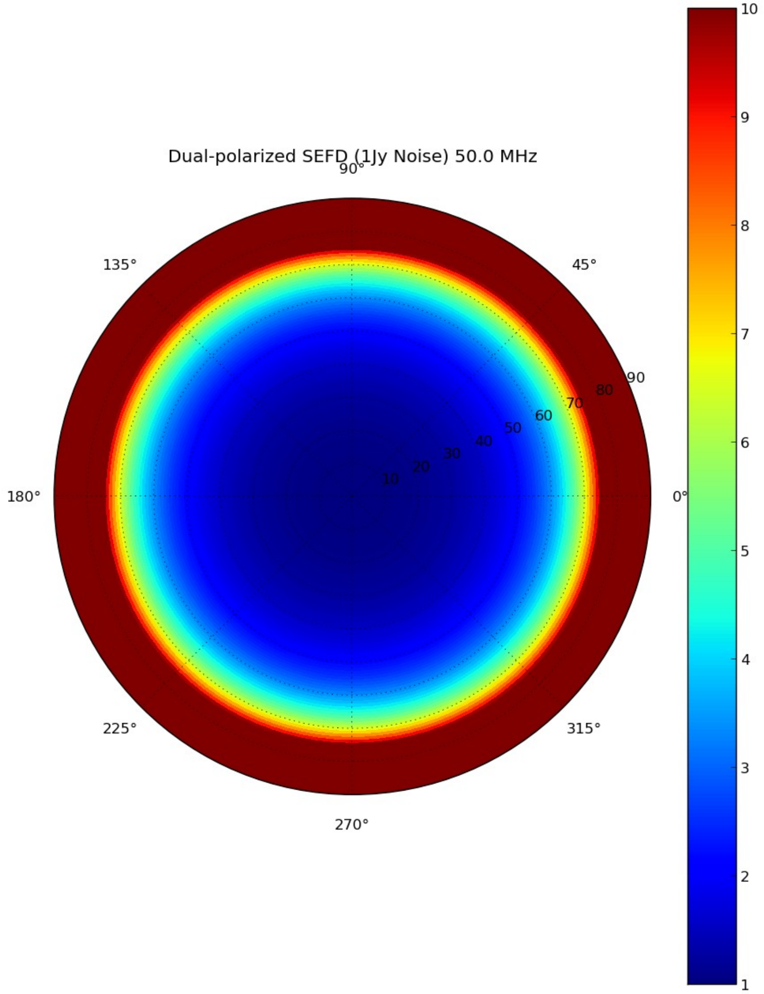
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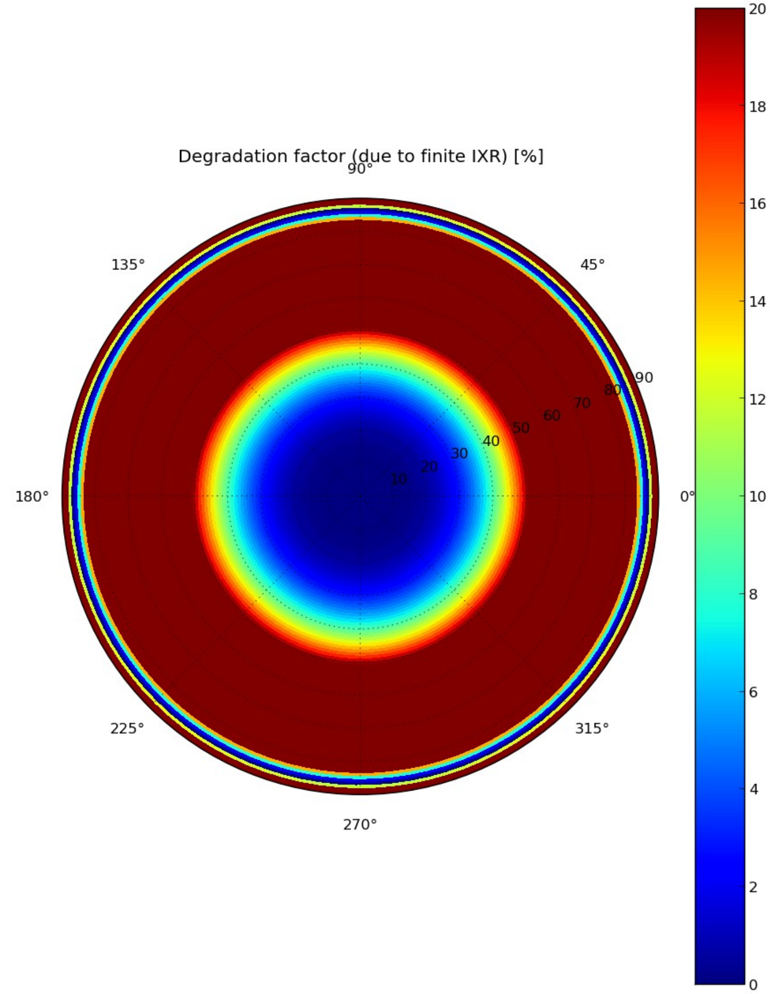
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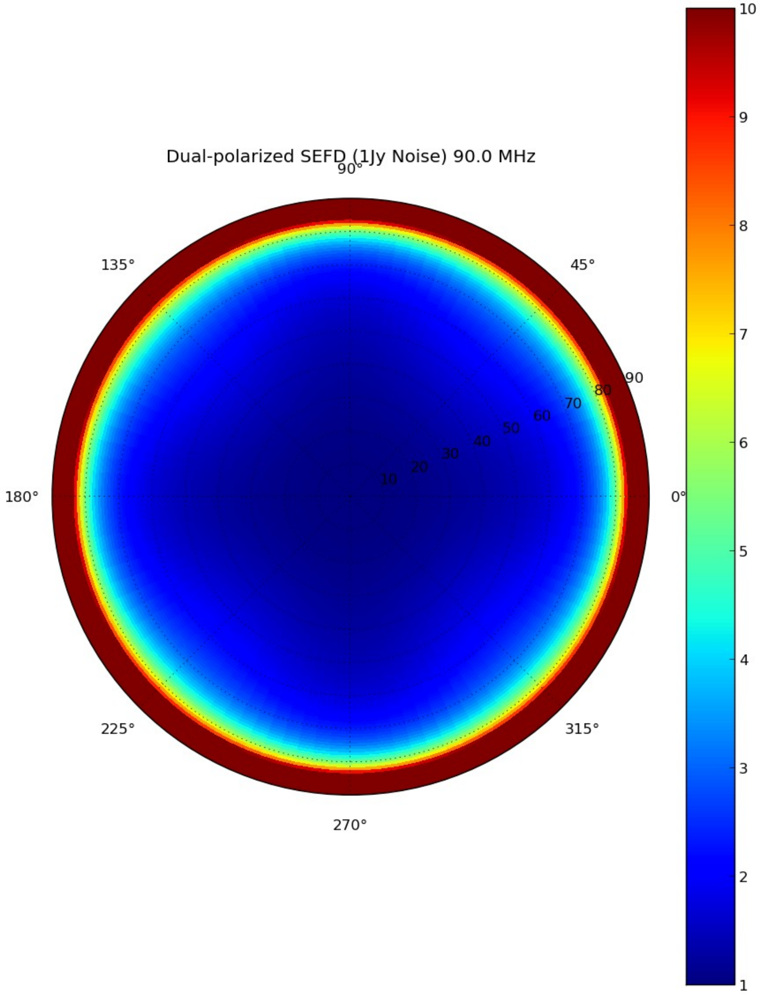
90 MHz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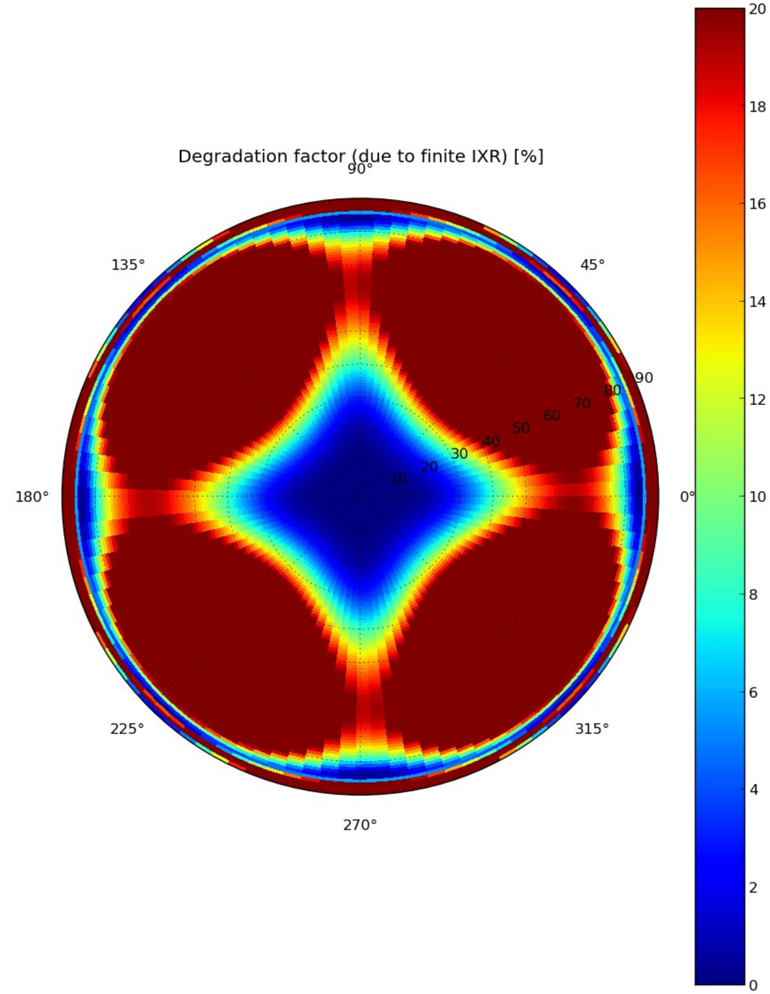
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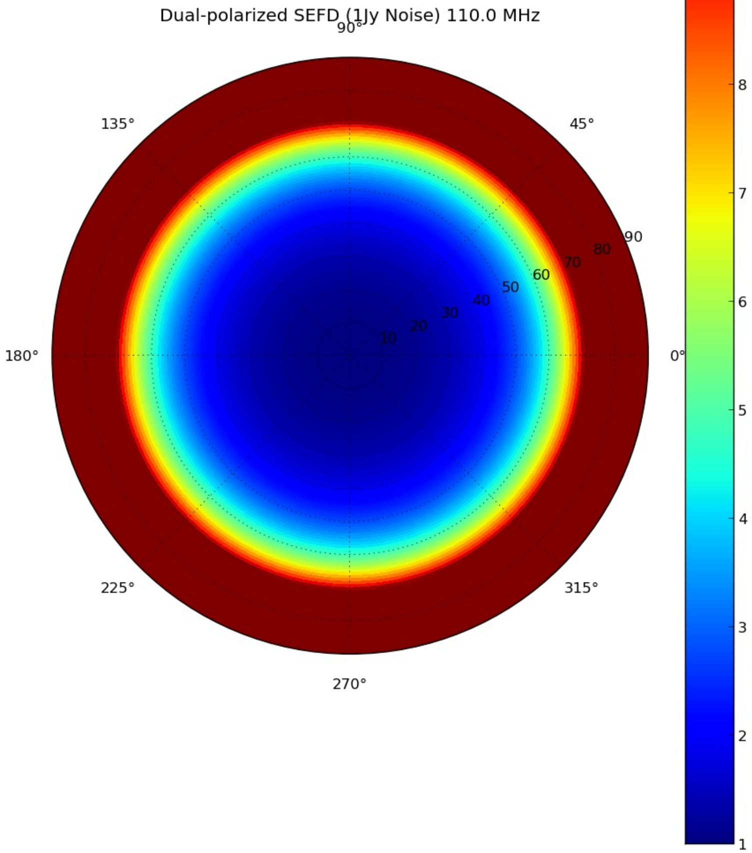
110 MHz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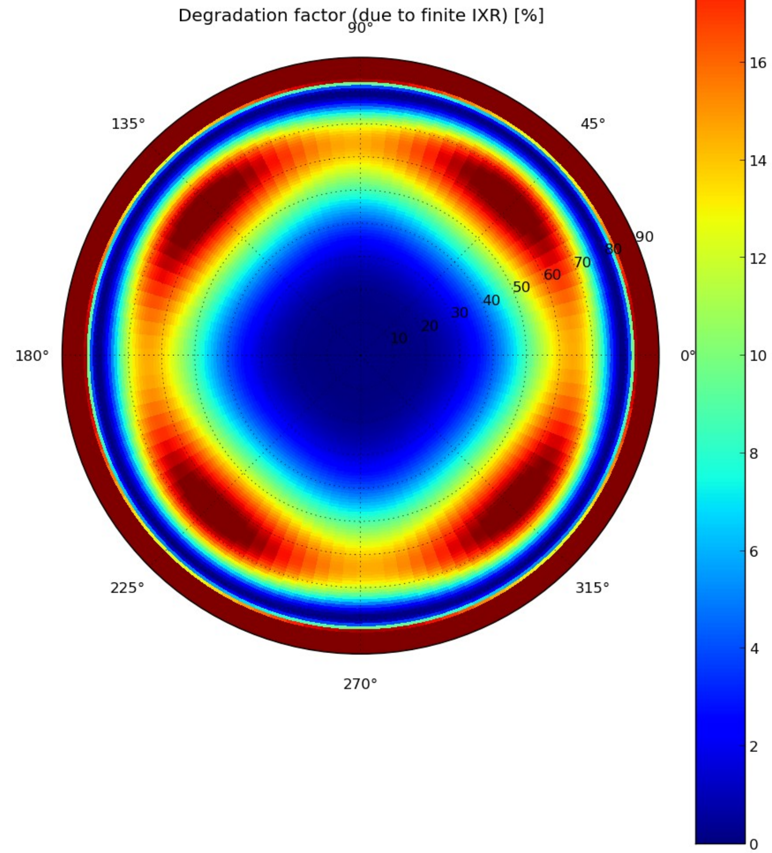
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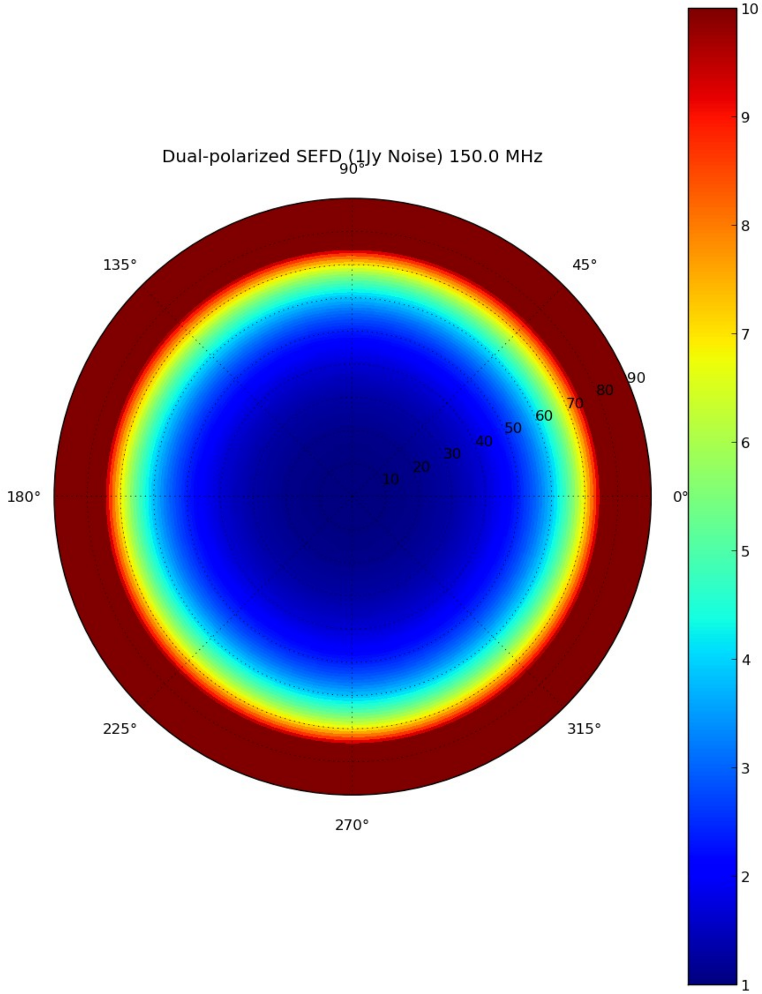
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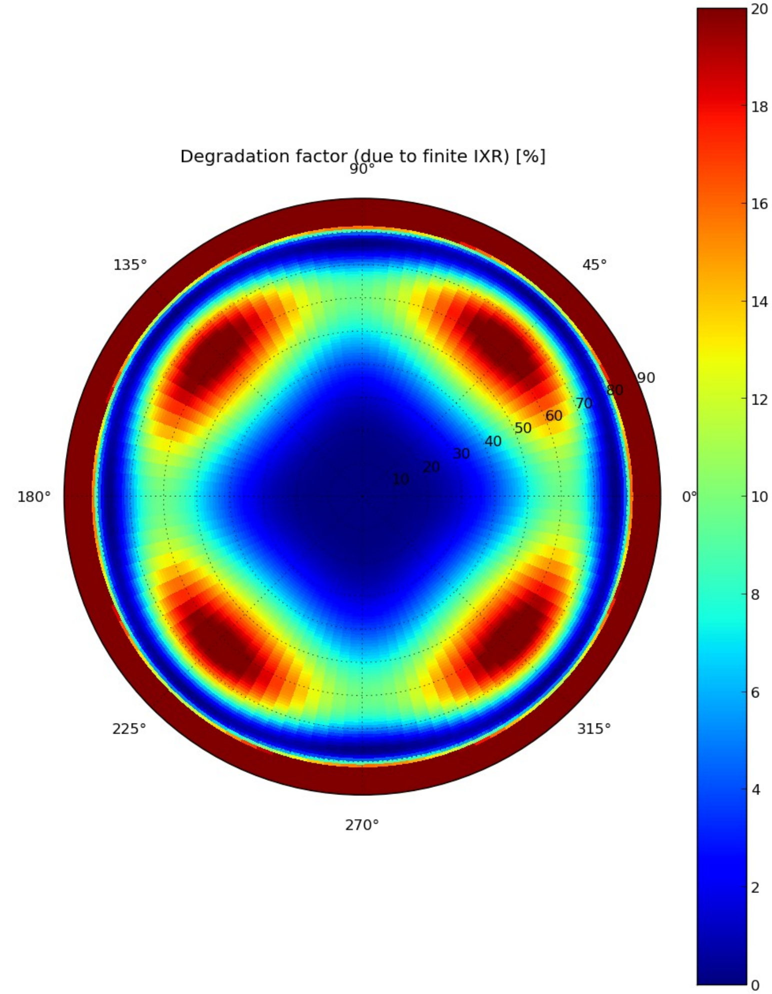
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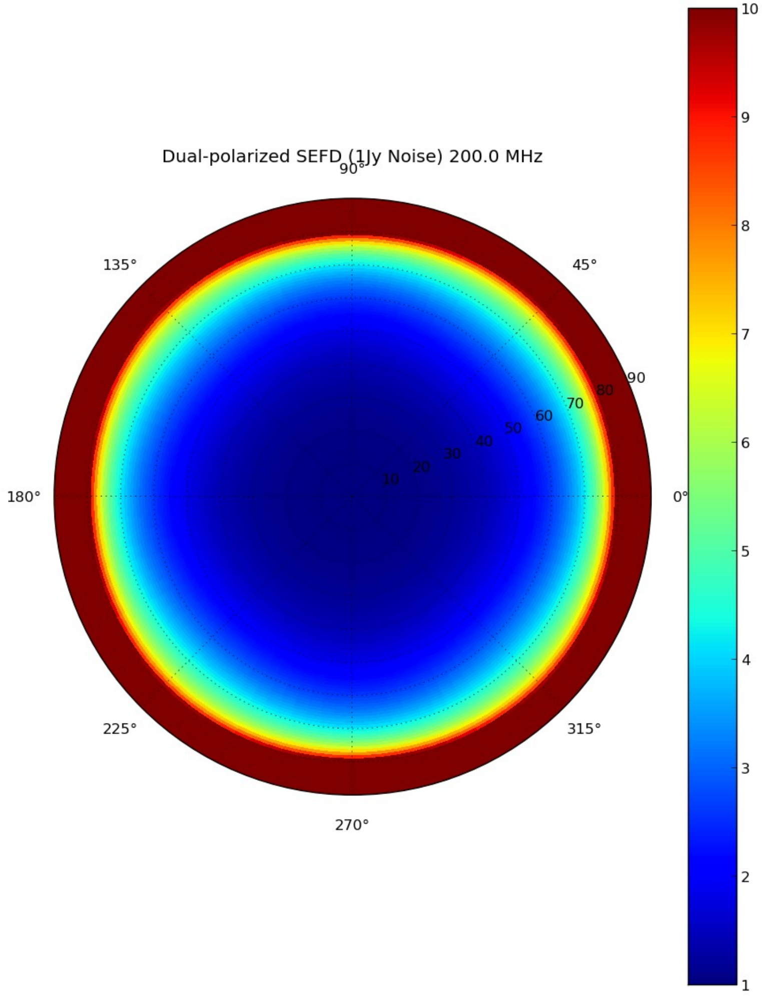
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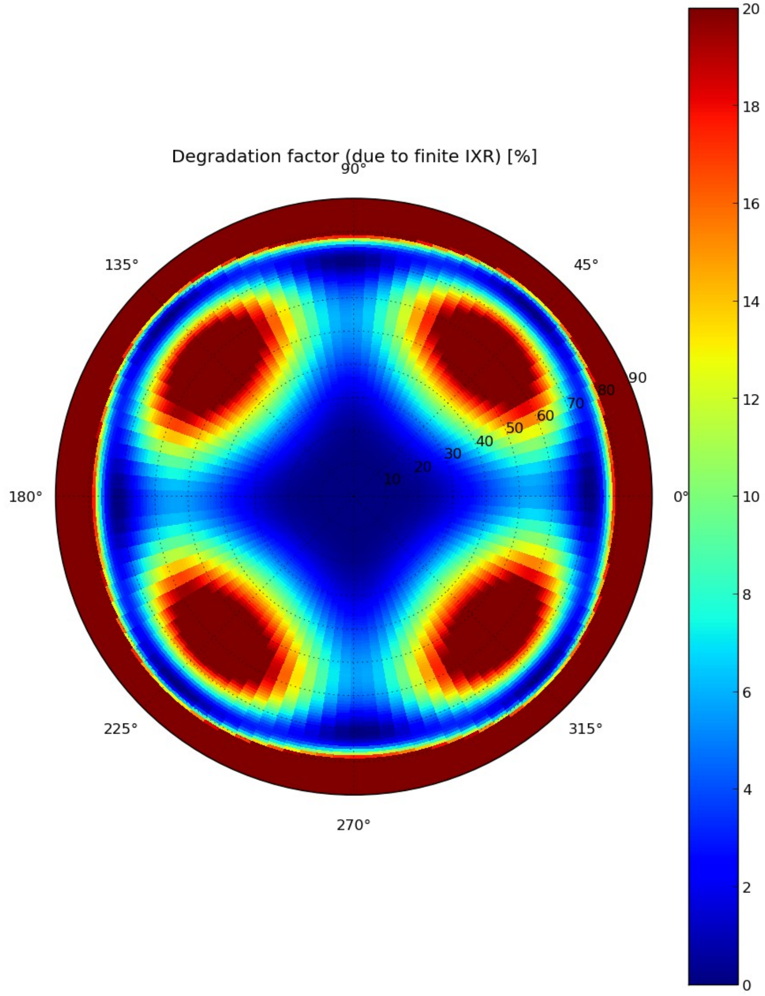
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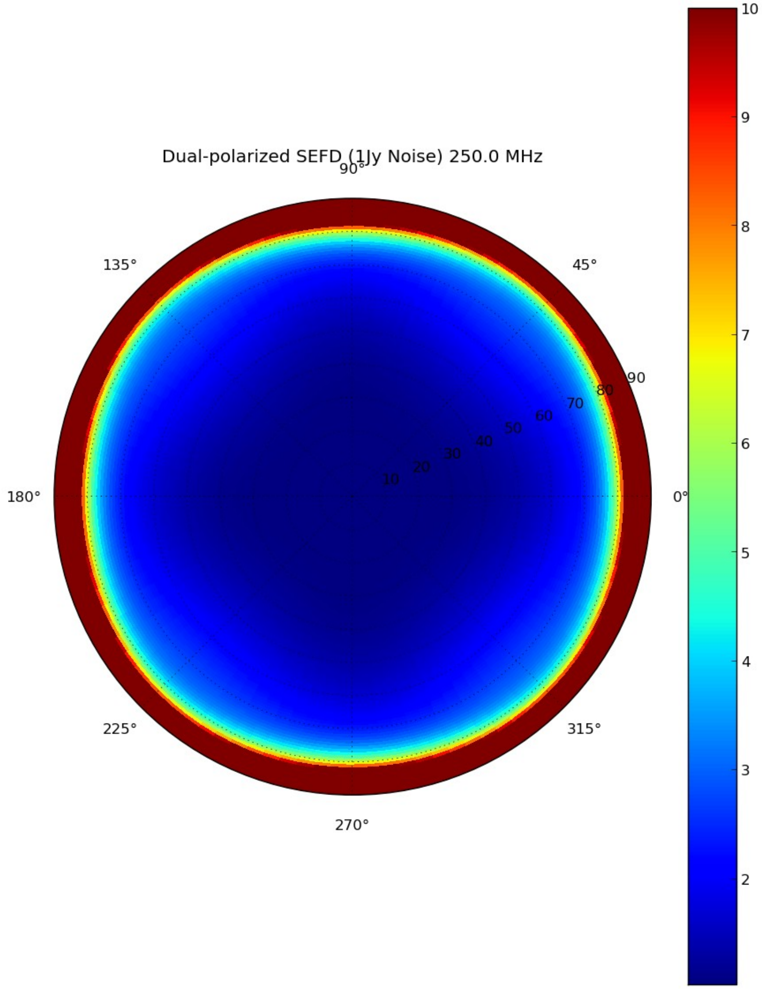
250 MHz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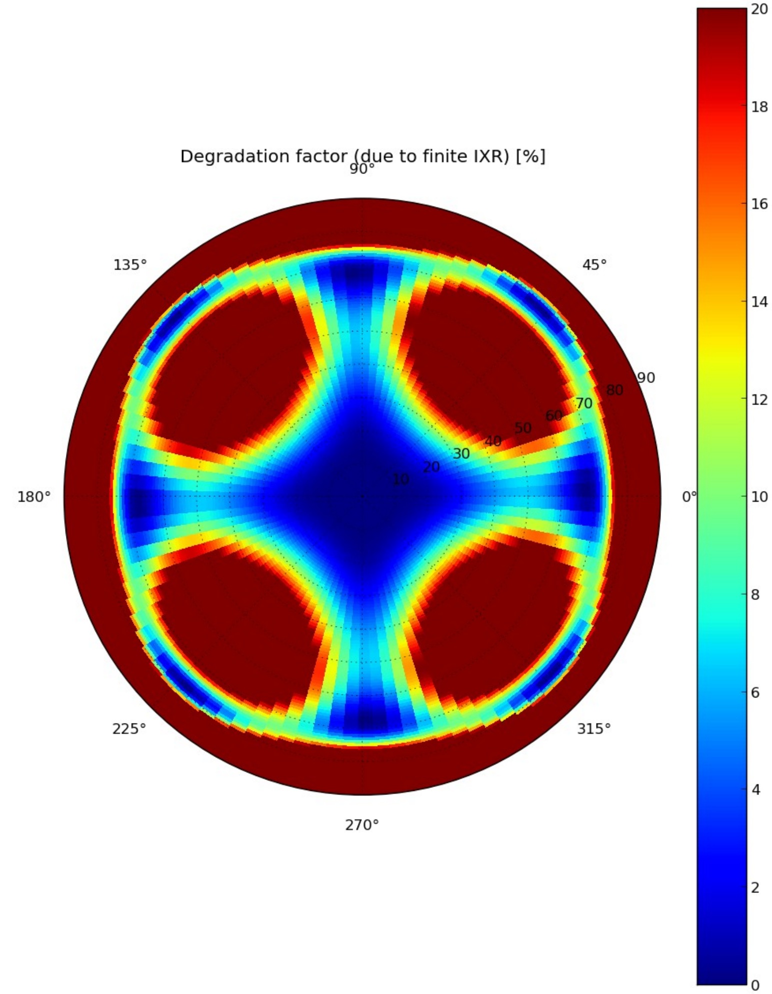
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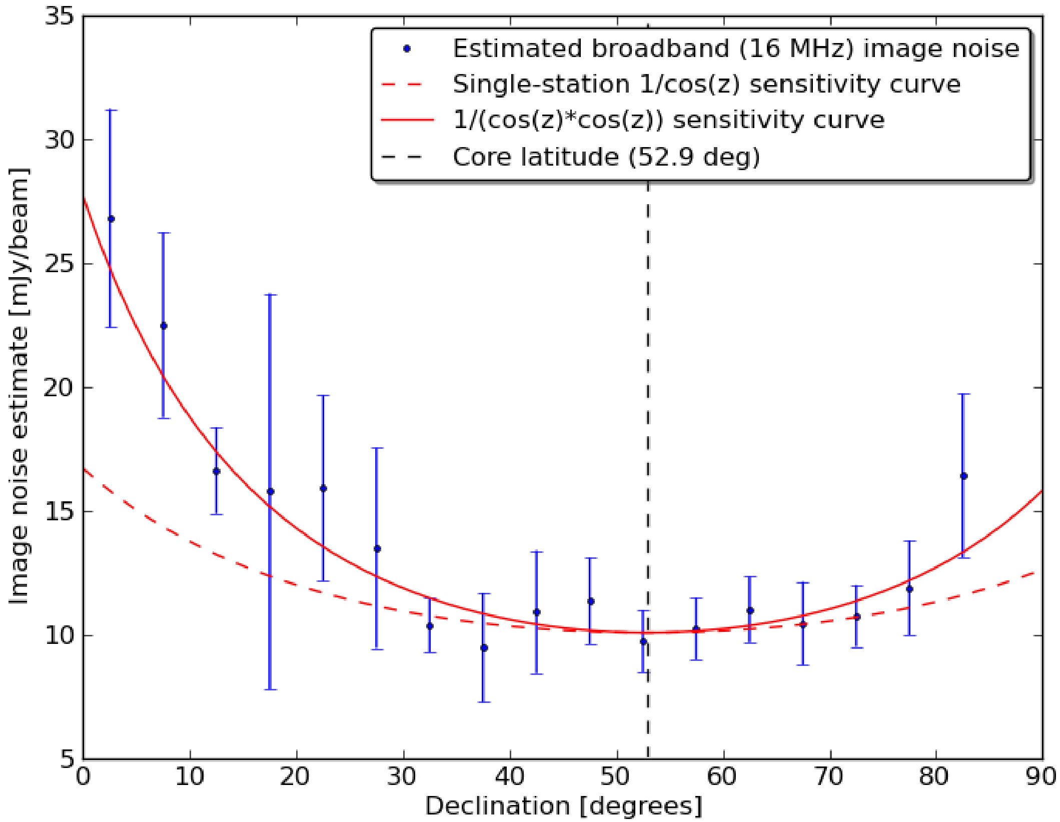
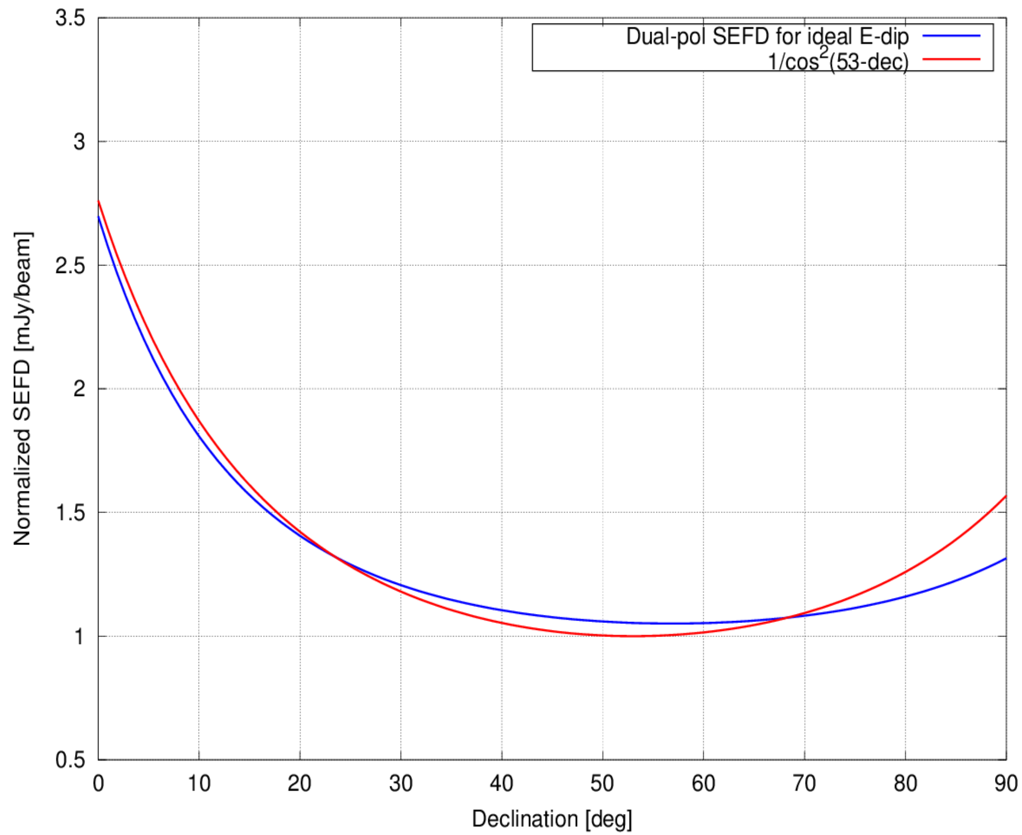
MSSS image noise model?
...Is a Lofar station a Lambertian sink?



Courtesy George Heald



SEFD for ideal electric dipole

$$n = \sin(el)$$

$$IXR_M(edip) = \frac{n^2 + 1}{n^2 - 1}$$

$$G(edip) = \frac{n^2 + 1}{2}$$

$$SEFD = \frac{\sqrt{1 + n^4}}{\sqrt{2}\, n^2}$$

Conclusions & Future

- SEFD for dual-polarized antennas with finite polarization purity has been found
- Lofar sensitivity has elevation dependency
  - Increases by about 3 times at 45 degs relative zenith
  - and 5 times at 60 degs relative zenith
- SEFD for polarized intensity is currently under investigation