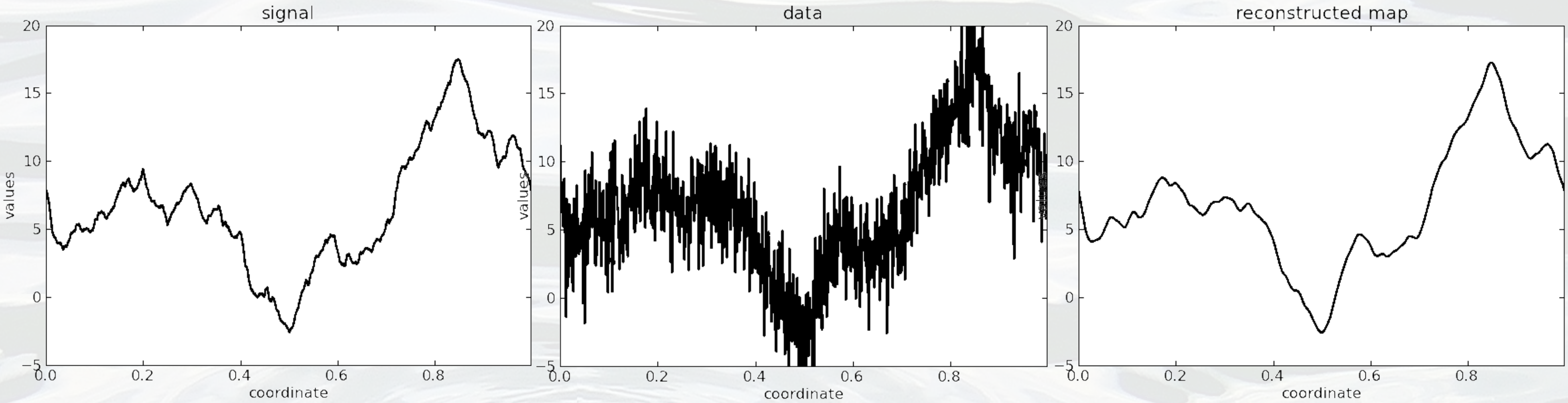
Information Field Theory

and its applicability
in radio astronomy & magnetic field studies

Torsten Enßlin – MPI for Astrophysics

Information Field Theory



signal $s = s(x)$ is function over x-coordinate calibration(time), magnetic field(position), etc.

data d responds to signal, ideally in a linear way

$$d = R\,s + n, \quad d_i = \int dx\, R_i(x)\, s(x) + n_i$$

reconstruction $m = F(d)$

Information Field Theory

signal field: ∞ degrees of freedom
data set: finite
→ additional information needed

information:
physical laws, symmetries, continuity, statistical homogeneity/isotropy, ...

combining concrete (data) & abstract (knowledge) information
→ **information theory for fields**

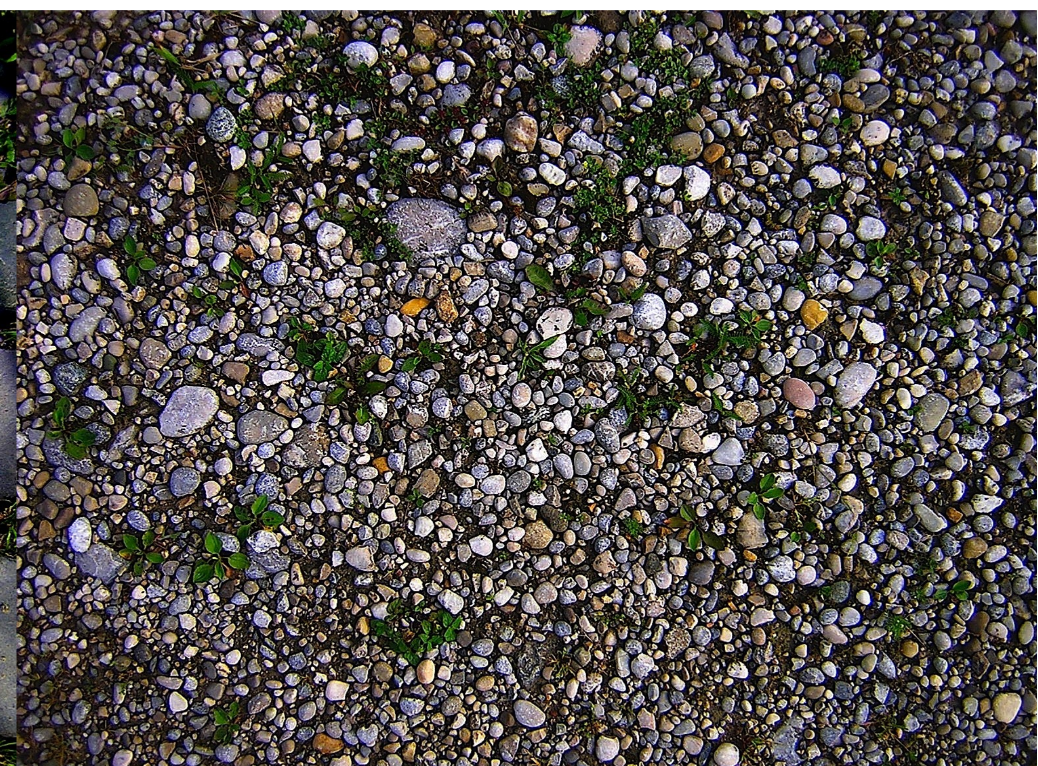
$$\mathcal{P}(s|d) = \frac{\mathcal{P}(d|s)\,\mathcal{P}(s)}{\mathcal{P}(d)}$$

$$s = \text{signal field}$$

$$d = \text{data}$$

$$H(d,s) = -\log \mathcal{P}(d,s)$$

$$Z(d) = \int \mathcal{D}s\, \mathcal{P}(d,s)$$



Free Theory

Gaussian signal & noise, linear response

signal :

$$
\begin{aligned}
P(s) &= \mathcal{G}(s,S) = \frac{1}{|2\pi\, S|^{1/2}} \exp\left(-\frac{1}{2}\, s^\dagger S^{-1} s\right) \\
j^\dagger s &= \int dx\, j^*(x) s(x) \\
S &= \langle s\, s^\dagger \rangle_{(s)}
\end{aligned}
$$

data :

$$
d = R\, s + n,\ P(d|s) = P(n = d - R\, s)
$$

noise :

$$
P(n) = \mathcal{G}(n, N),\ N = \langle n\, n^\dagger \rangle_{(n)}
$$

Free Theory

Gaussian signal & noise, linear response

$$\text{Gauß} * \text{Gauß} = \text{Gauß}$$

Wiener filter theory

known for 60 years

$$H(d,s) = \frac{1}{2}(s-m)^\dagger D^{-1}(s-m) + \text{const}$$

posterior distribution: $\mathcal{P}(s|d) = \mathcal{G}(s-m, D)$

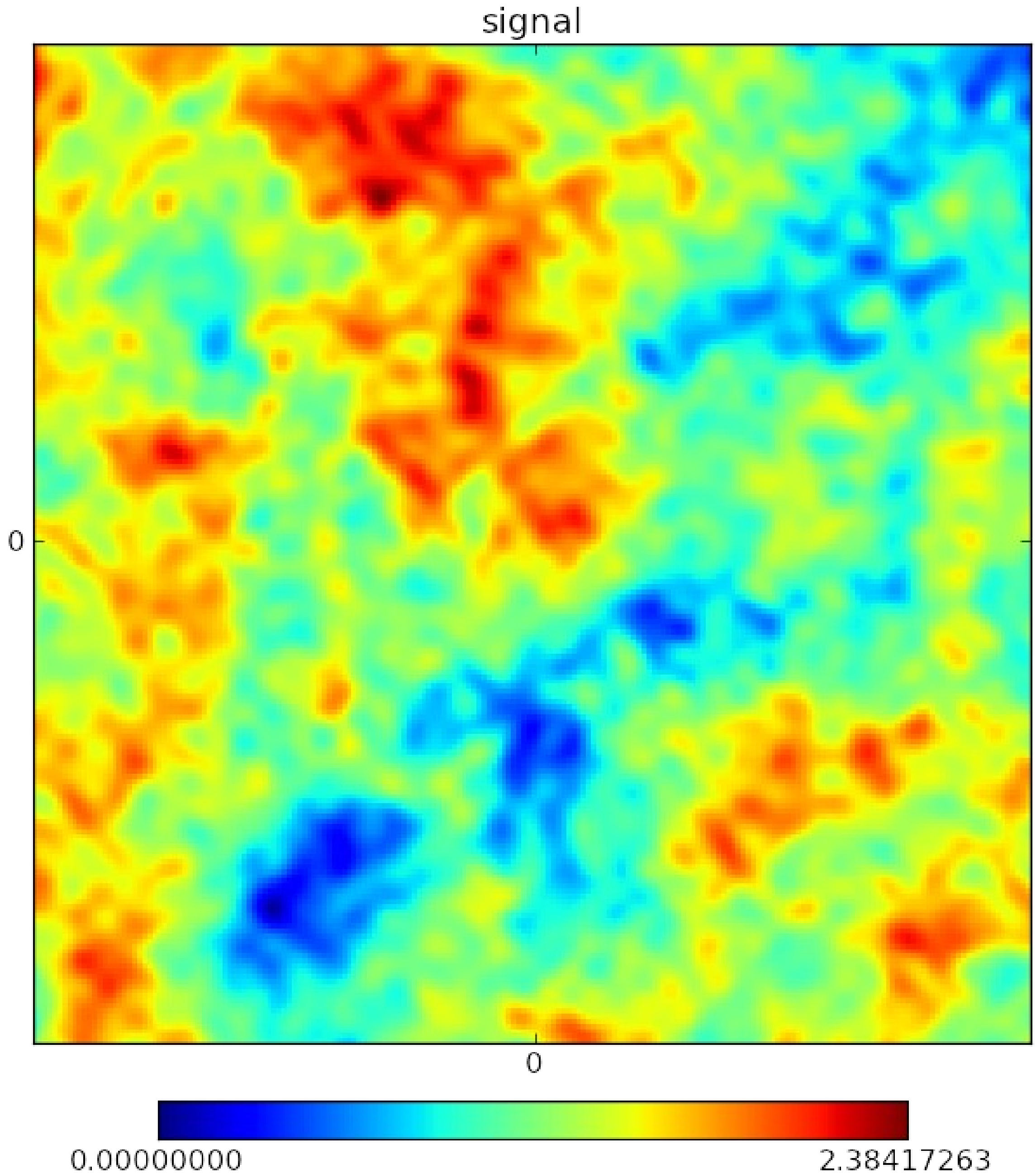
a posteriori mean: $m = D\,j$

information propagator: $D = \left(S^{-1} + R^\dagger N^{-1} R\right)^{-1}$

information source: $j = R^\dagger N^{-1} d$

$$m = (S^{-1} + R^\dagger N^{-1} R)^{-1} R^\dagger N^{-1}\, d$$

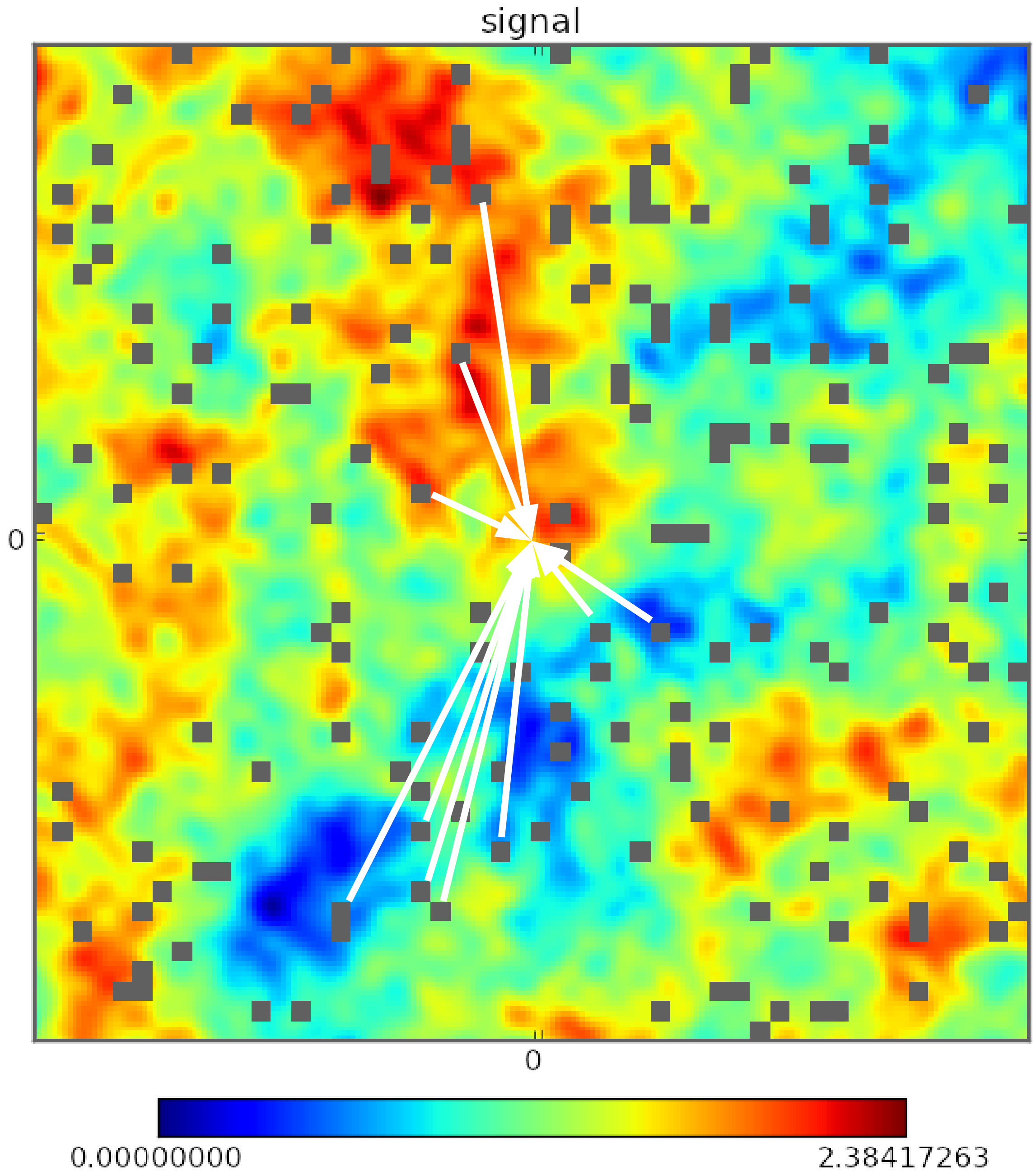
$$m(k) = \frac{d(k)}{1 + P_n(k)/P_s(k)} \quad \text{for } R = \text{id}$$



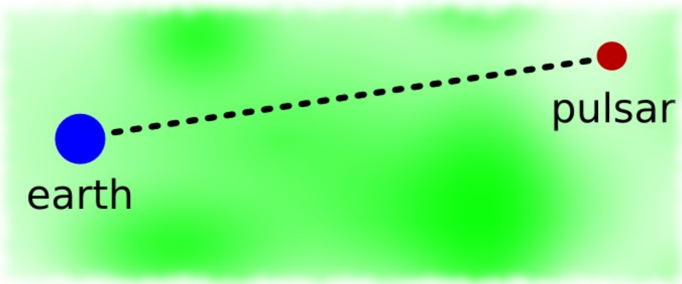
signal field: electron density

mock signal from known power spectrum

$$P_s(k) = P_0 \left(1 + \frac{k^2}{k_0^2}\right)^{-\frac{4}{3}}$$



data = integrated signal



$$d = R\,s + n$$

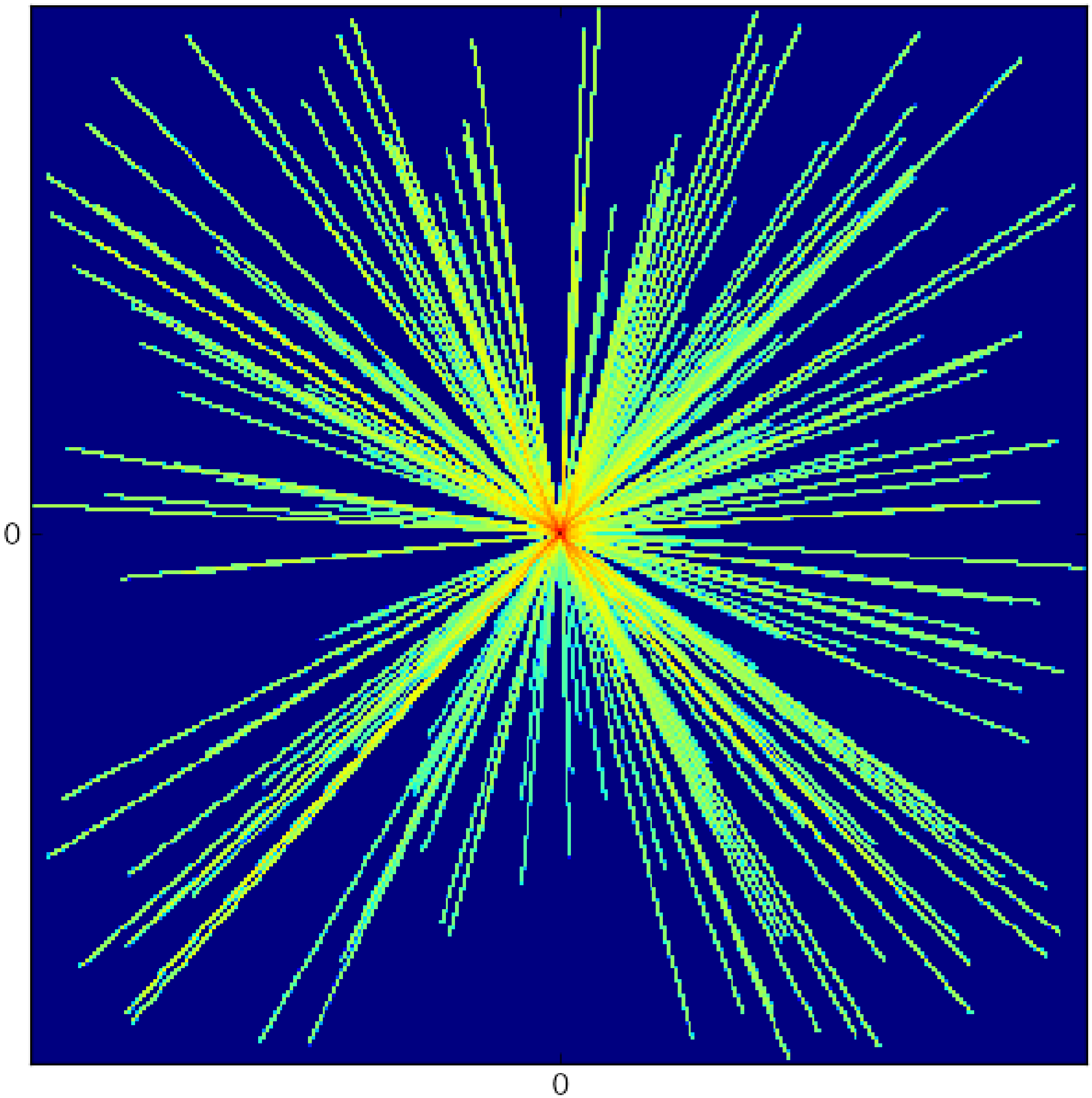
$$(Rs)_i = \int_{\text{Earth}}^{\text{pulsar } i} dz\; s(z)$$

$$j = R^\dagger\, N^{-1}\, d$$

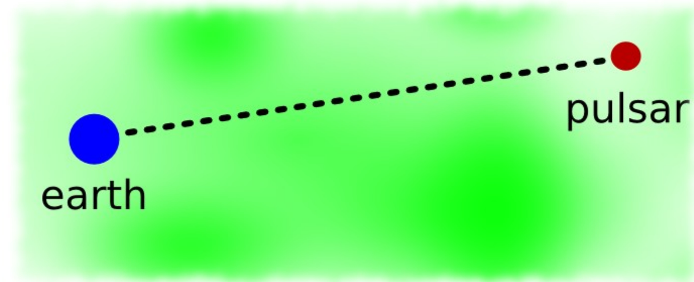
$$D = \left(S^{-1} + R^\dagger N^{-1} R\right)^{-1}$$

$$m = D\, j$$

information source



j



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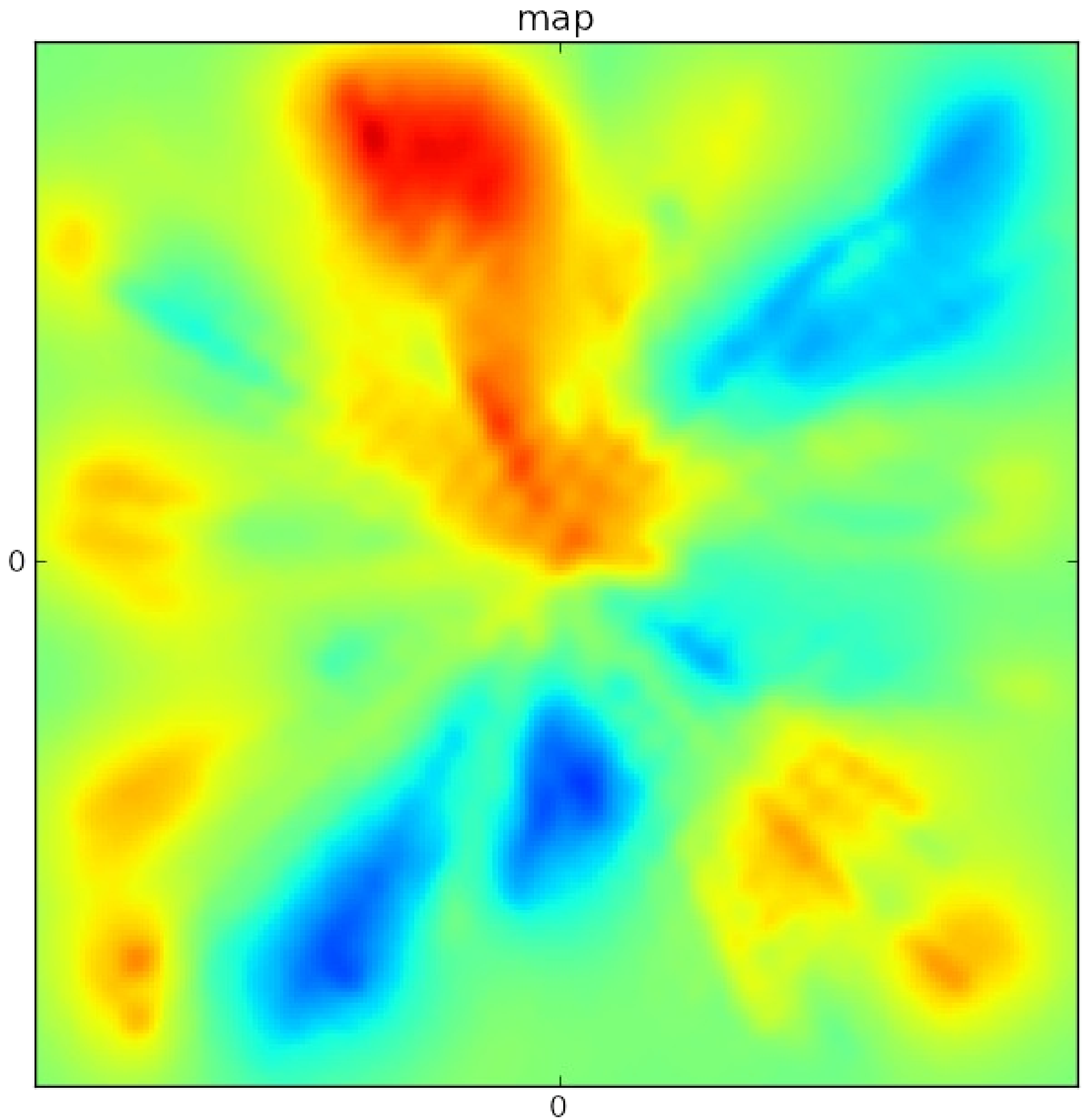
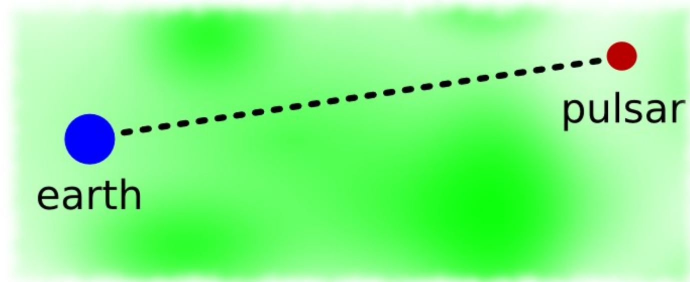
reconstruction



m



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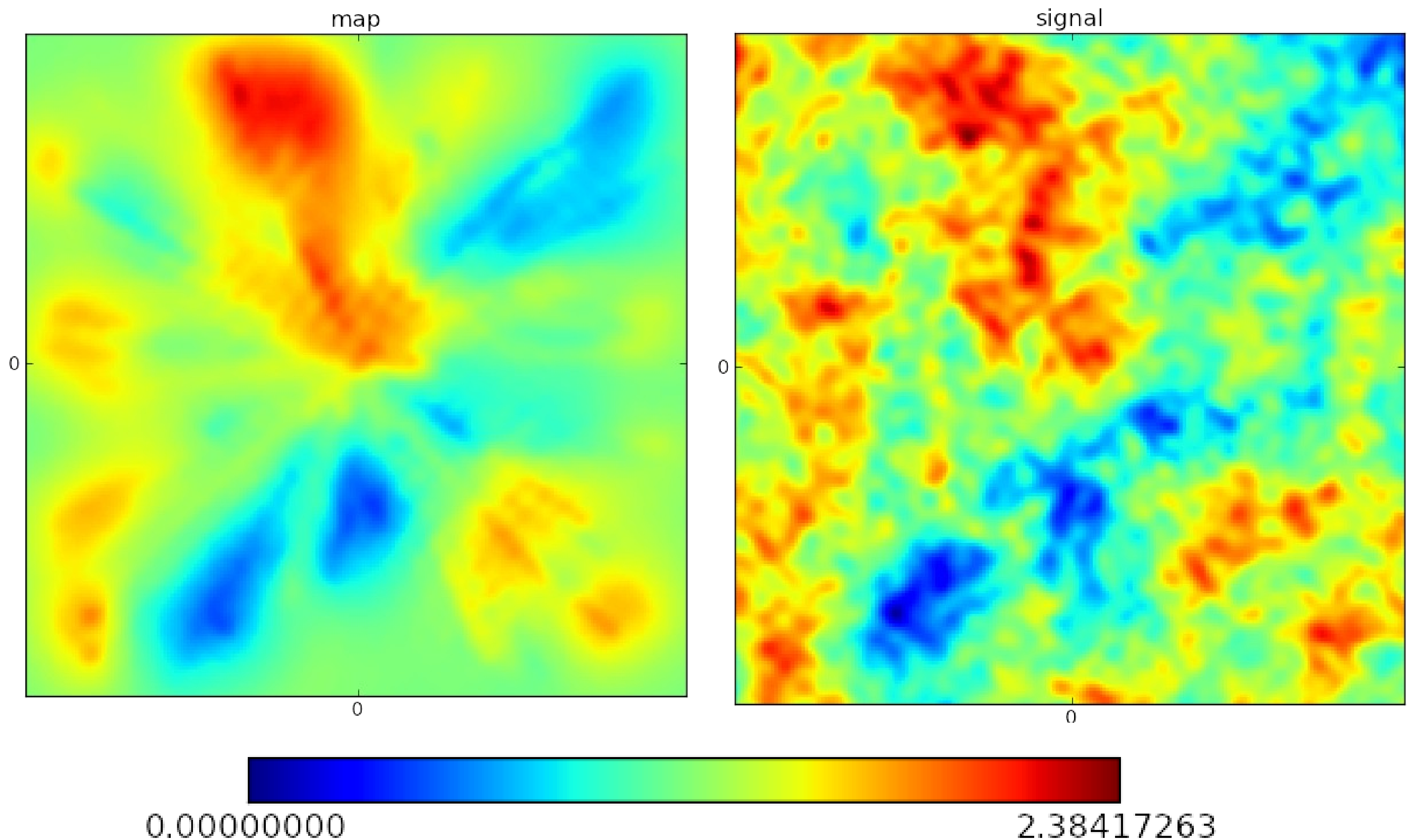


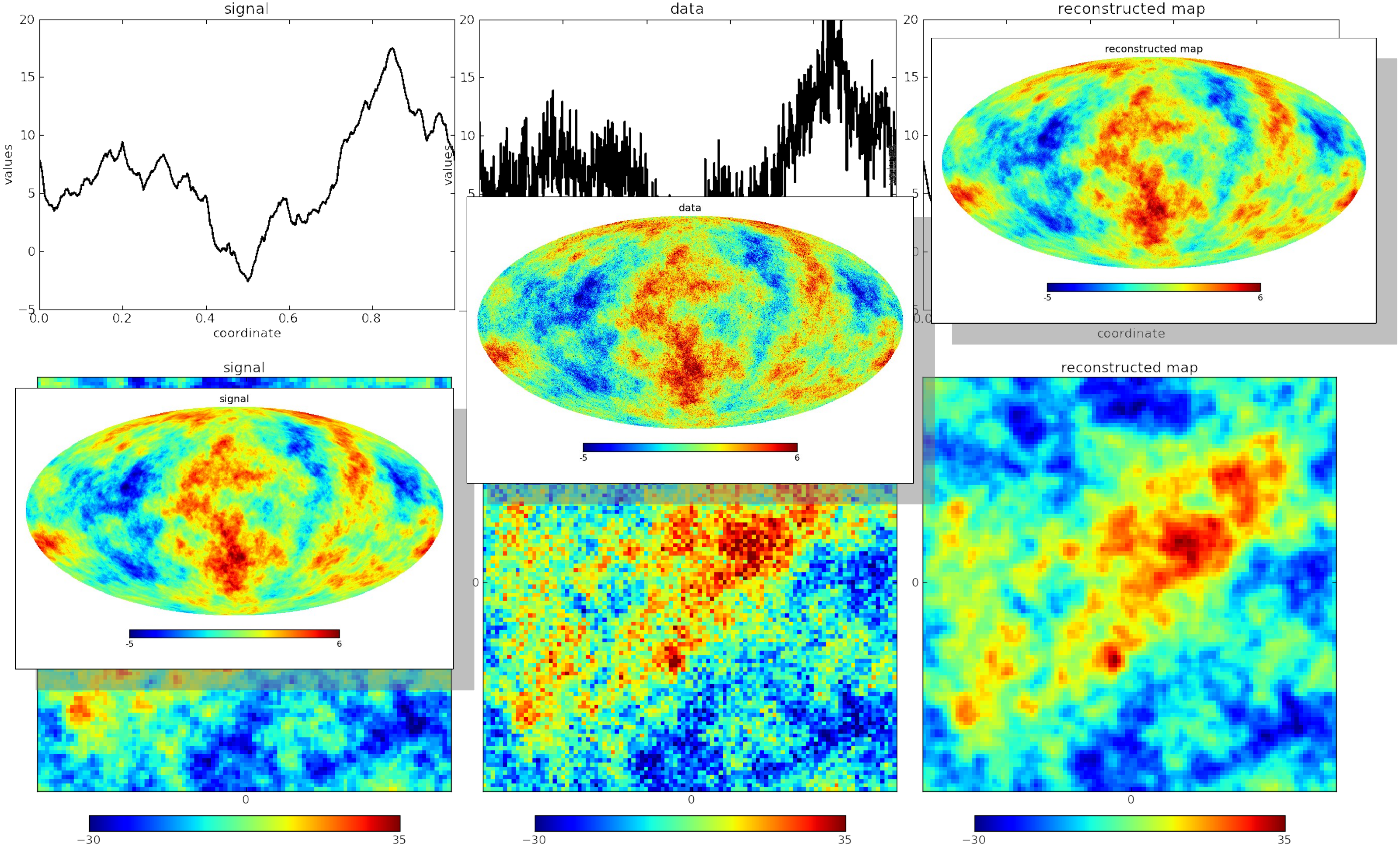
map
signal
0
0
0
0
0.00000000
2.38417263

NIFTY – Numerical Information Field Theory

$m = D\,j$

Selig et al. (arXiv:1301.4499)

Code & Docu @ http://www.mpa-garching.mpg.de/ift/nifty/



Extended Critical Filter

Gaussian signal & noise, but unknown covariances!

signal covariance: $S_{xy} = \langle s_x \, s_y^\dagger \rangle_{(s)} = C_s(x-y)$

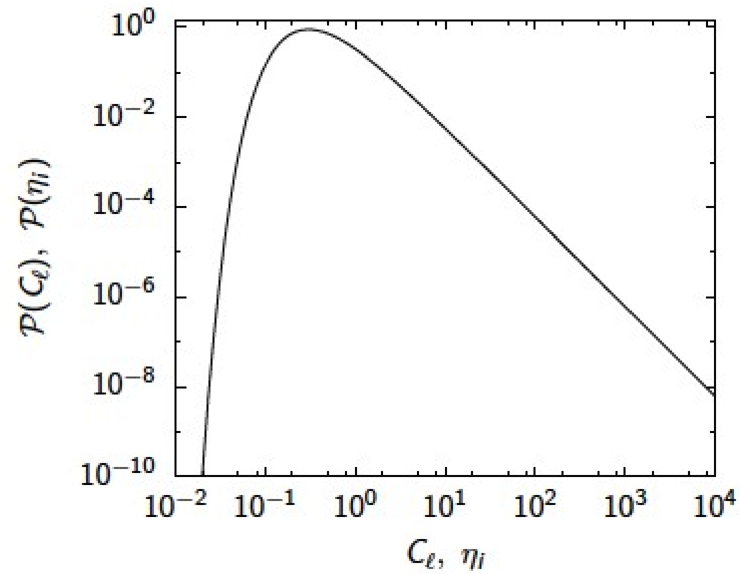
noise covariance: $N_{ij} = \langle n_i \, n_j^\dagger \rangle_{(n)} = \sigma_i^2 \, \delta_{ij}$

$$S = \sum_{k=0}^{k_{\text{max}}} p_k S_k \qquad N = \sum_{i=0}^{i_{\text{max}}} \eta_i N_i$$

assume priors for parameters

$$\mathcal{P}\left((p_k)_k\right) = \prod_{k=0}^{k_{\text{max}}} \frac{1}{q_k \Gamma(\alpha_k - 1)} \left(\frac{p_k}{q_k}\right)^{-\alpha_k} \exp\left(-\frac{q_k}{p_k}\right)$$

$$\mathcal{P}\left((\eta_i)_i\right) = \prod_{i=0}^{i_{\text{max}}} \frac{1}{q_i \Gamma(\alpha_i - 1)} \left(\frac{\eta_i}{q_i}\right)^{-\alpha_i} \exp\left(-\frac{q_i}{\eta_i}\right)$$



Extended Critical Filter

Gaussian signal & noise, but unknown covariances!

$$m = Dj, \quad D = \left[\sum_k p_k^{-1} S_k^{-1} + \sum_i \eta_i^{-1} R^\dagger N_i^{-1} R\right]^{-1},$$

$$j = \sum_i \eta_i^{-1} R^\dagger N_i^{-1} d$$

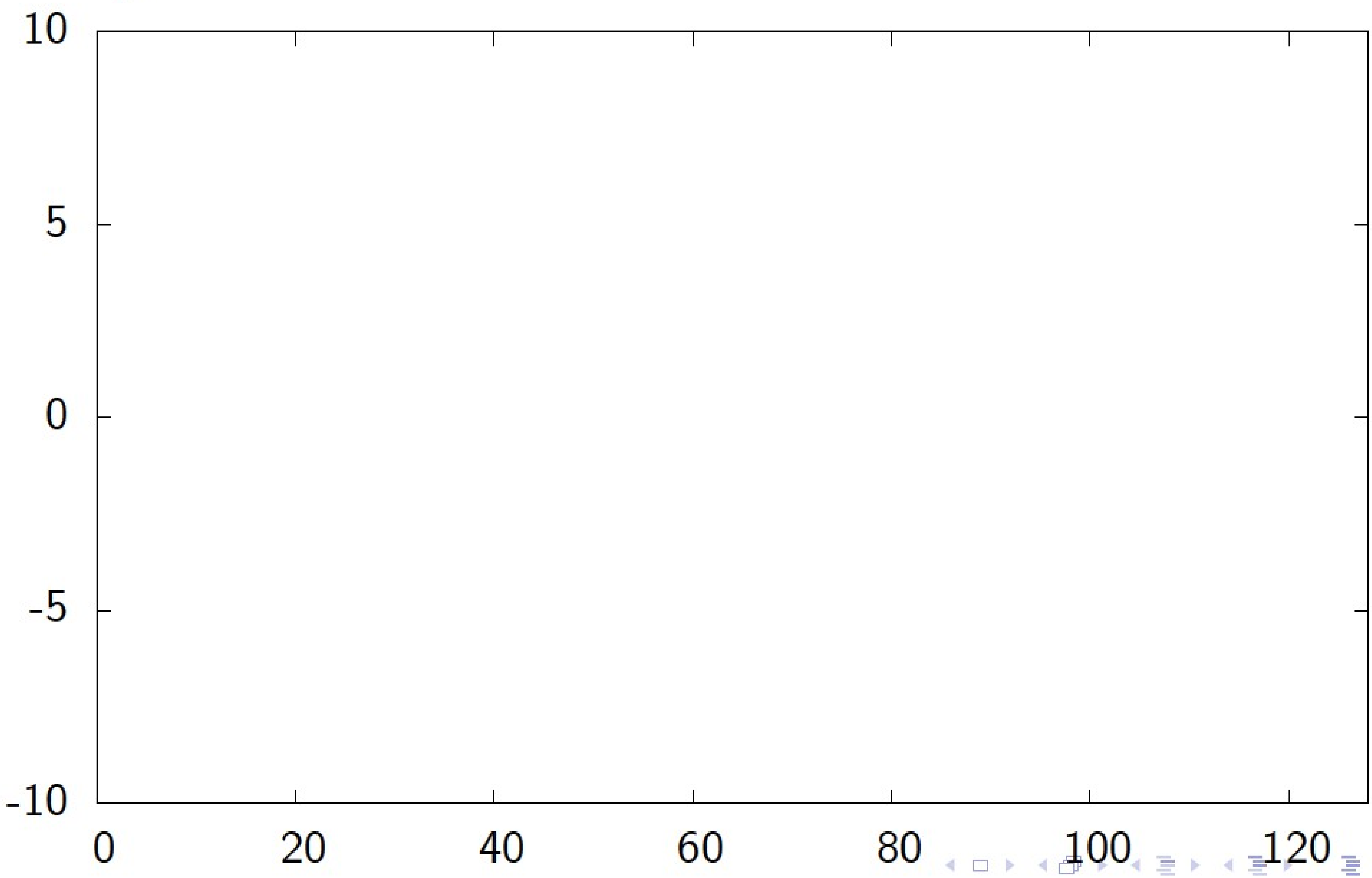
$$p_k = \frac{q_k + \frac{1}{2}\mathrm{tr}\left(\left(mm^\dagger + D\right) S_k^{-1}\right)}{\alpha_k - 1 + \mathrm{tr}\left(S_k S_k^{-1}\right)}$$

$$\eta_i = \frac{q_i + \frac{1}{2}\mathrm{tr}\left(\left((d - Rm)(d - Rm)^\dagger + RDR^\dagger\right) N_i^{-1}\right)}{\alpha_i - 1 + \mathrm{tr}\left(N_i N_i^{-1}\right)}$$

1D test case

Assumptions:

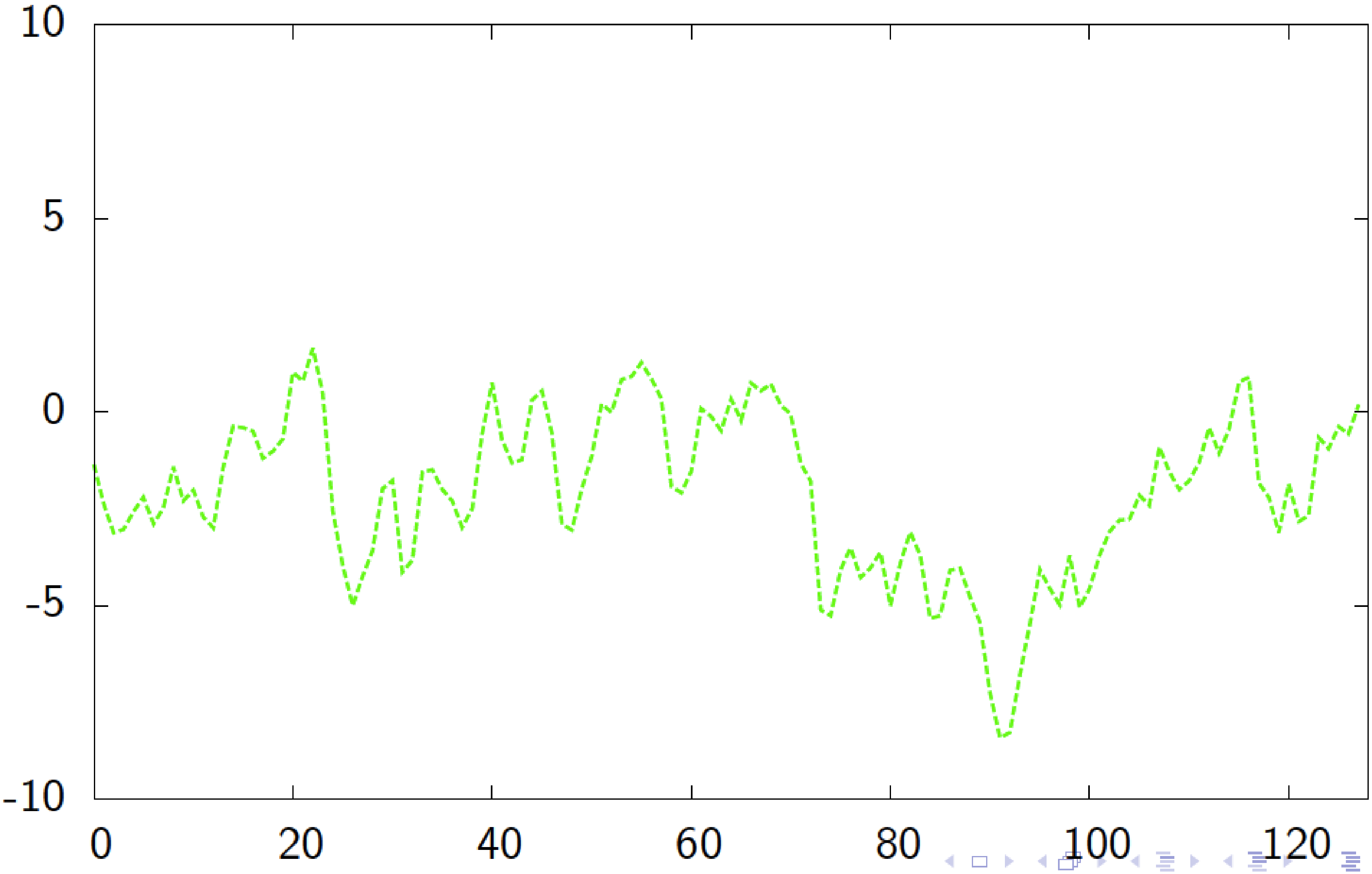
-
-



1D test case

Assumptions:

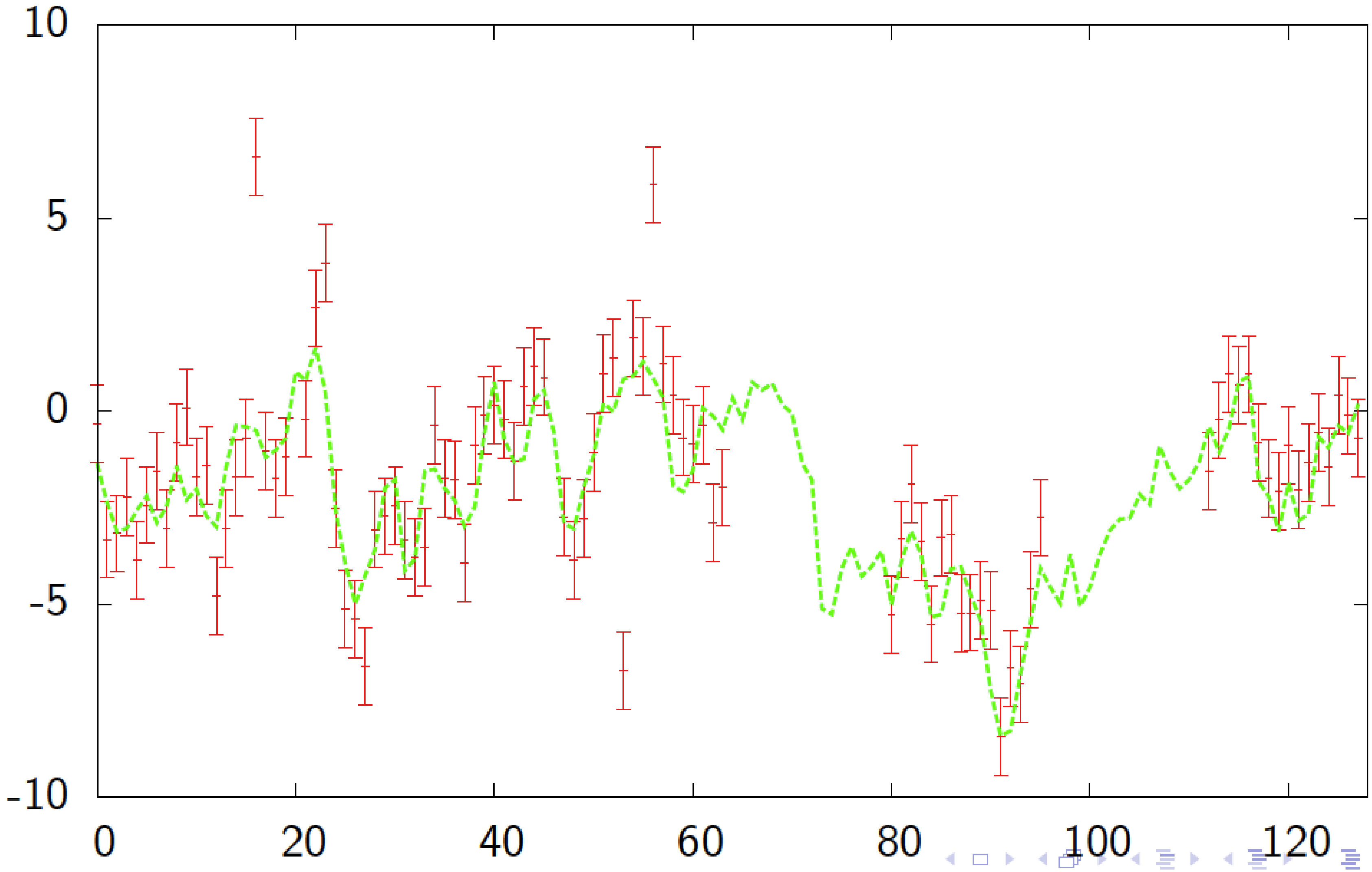
- signal field statistically homogeneous Gaussian random field
-



1D test case

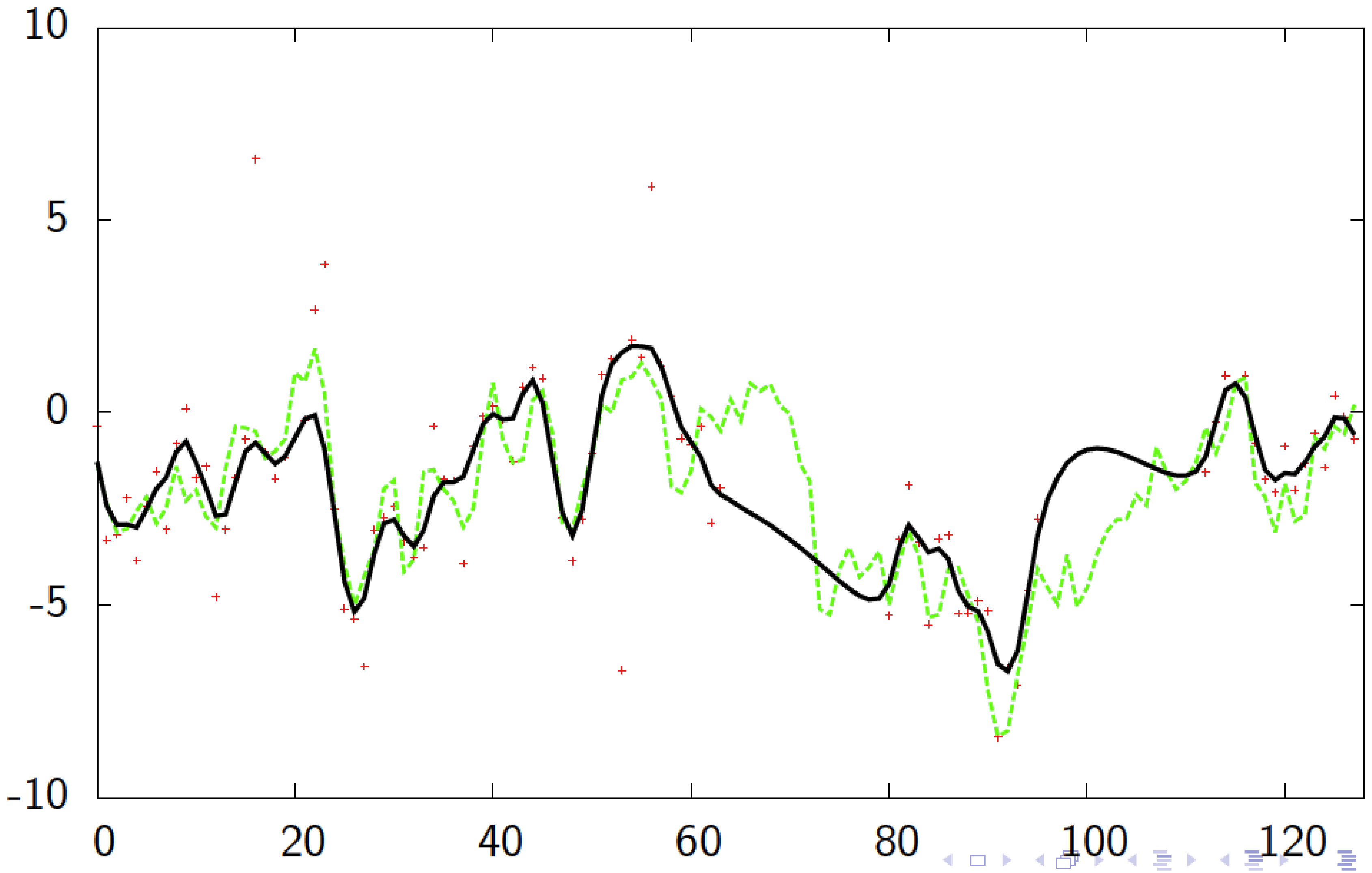
Assumptions:

- signal field statistically homogeneous Gaussian random field
- noise uncorrelated, Gaussian



1D test case

- Reconstruct (iteratively):
  signal, power spectrum, noise variance



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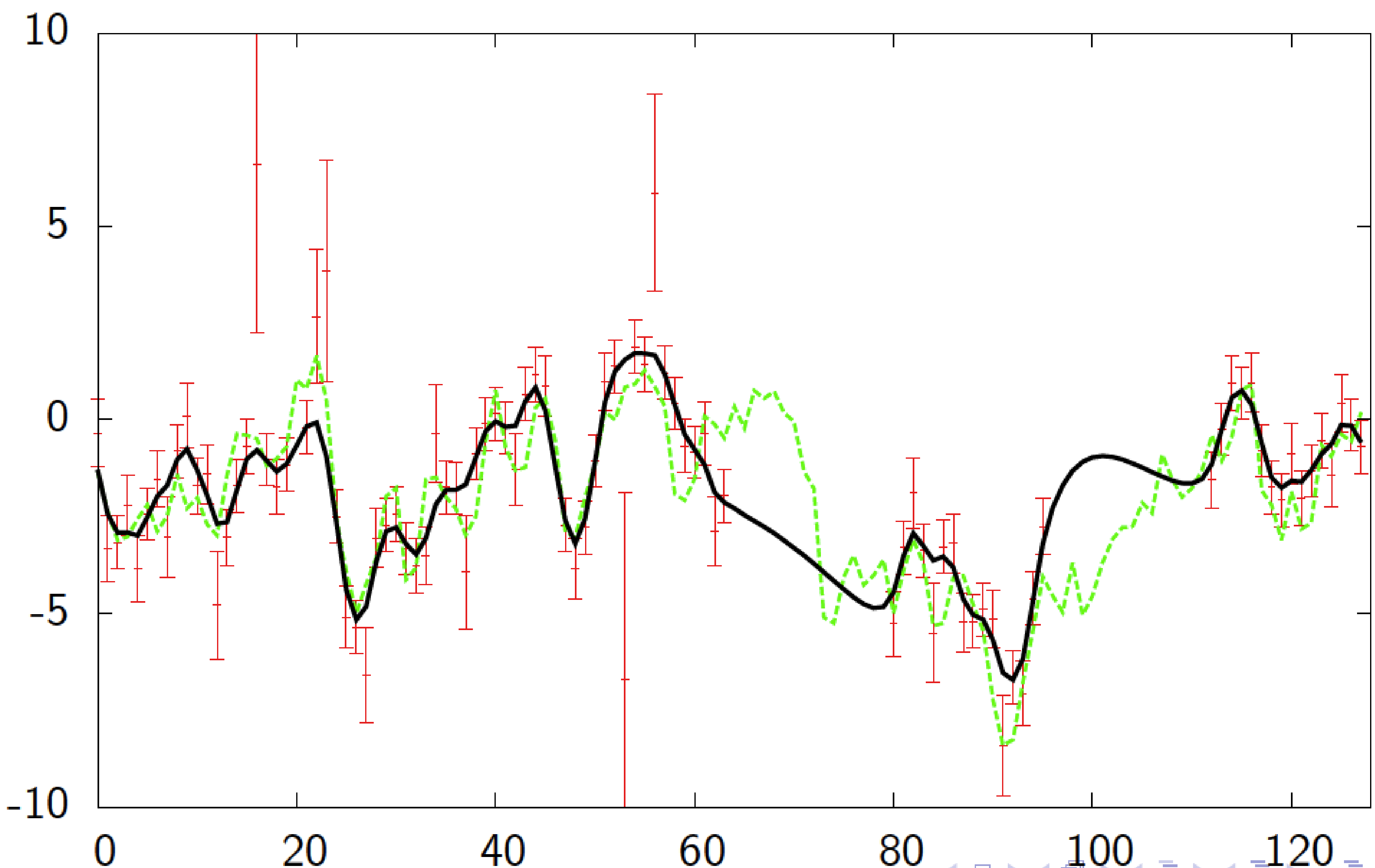
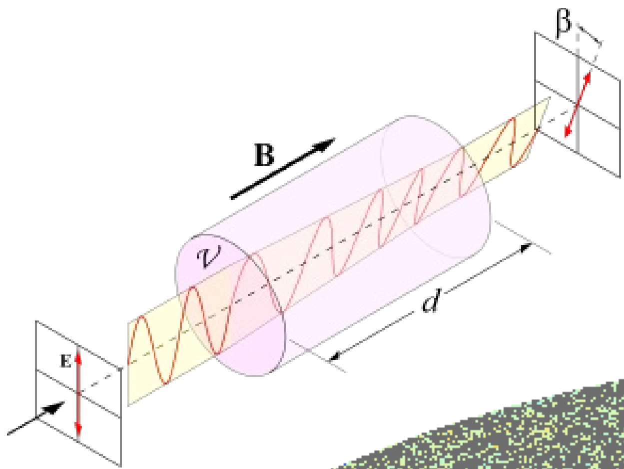


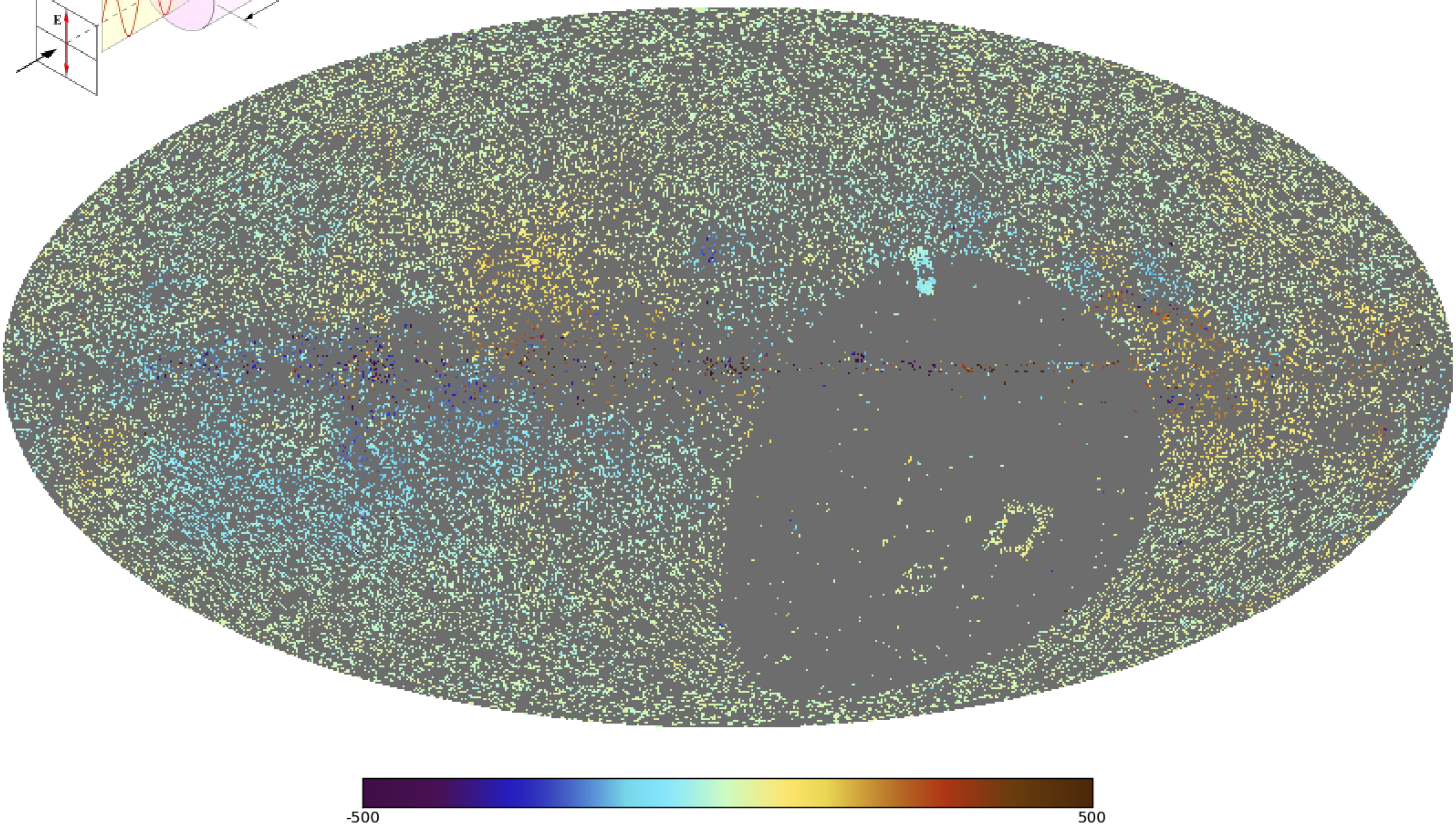


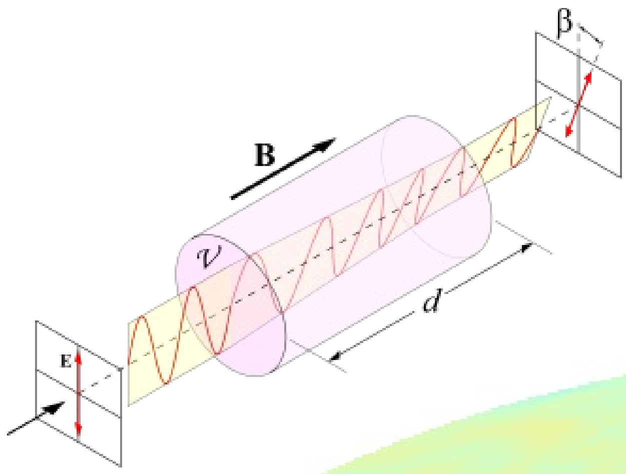
Faraday Sky

Oppermann et al. (2012)

Faraday depth:

$$\phi(z) \propto \int_0^z dz\, n_\mathrm{e}\, B_z$$





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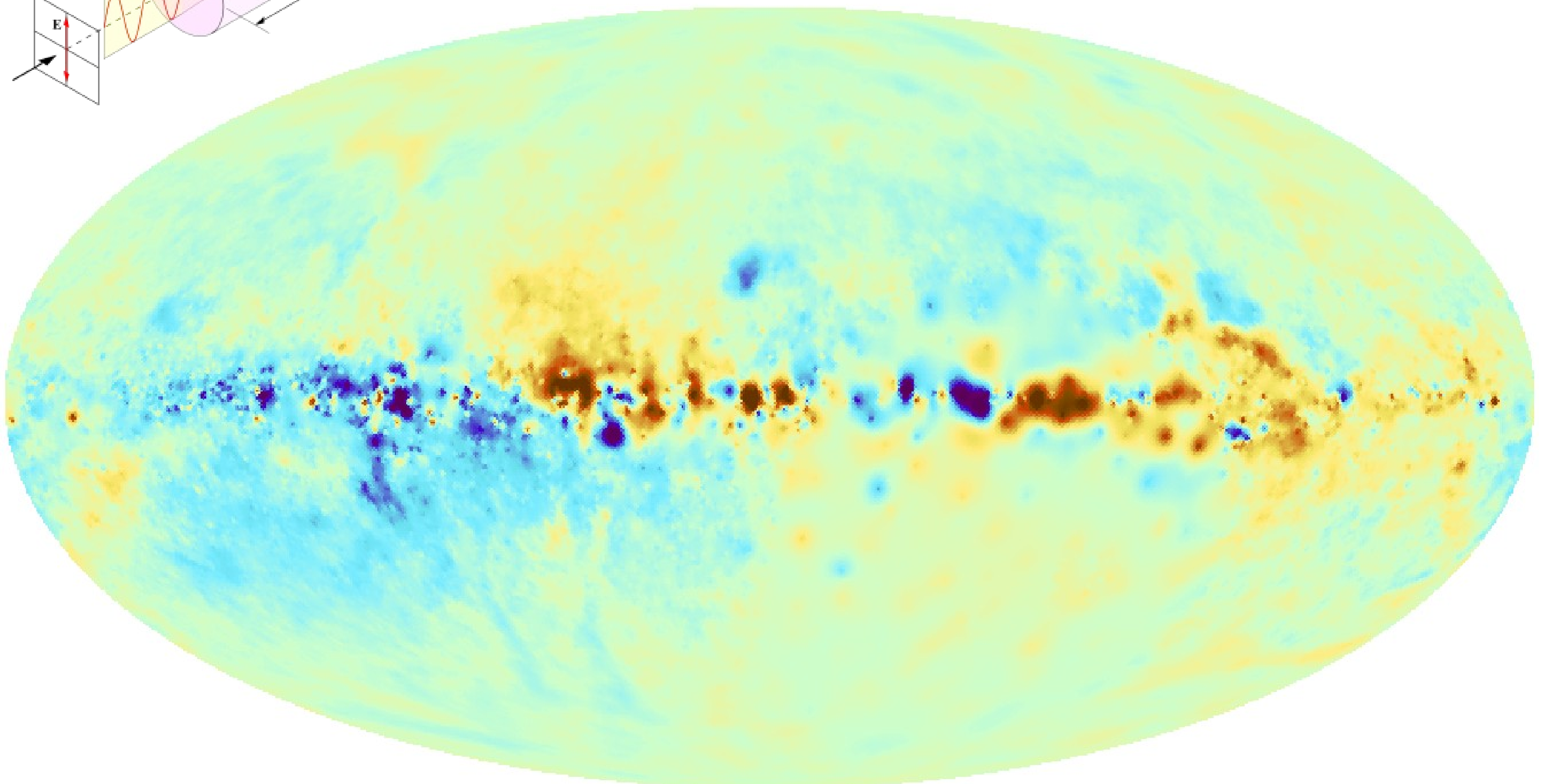

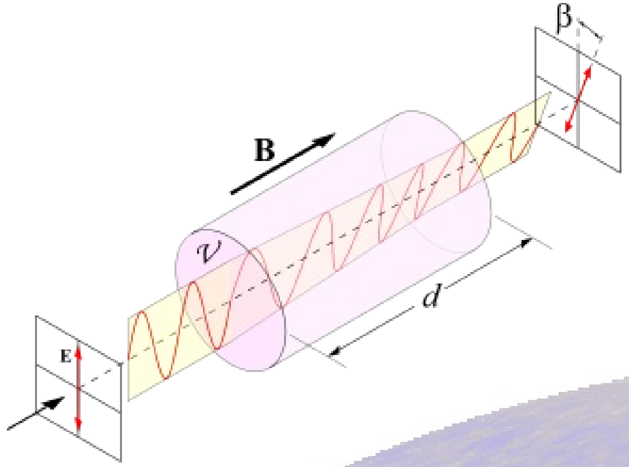

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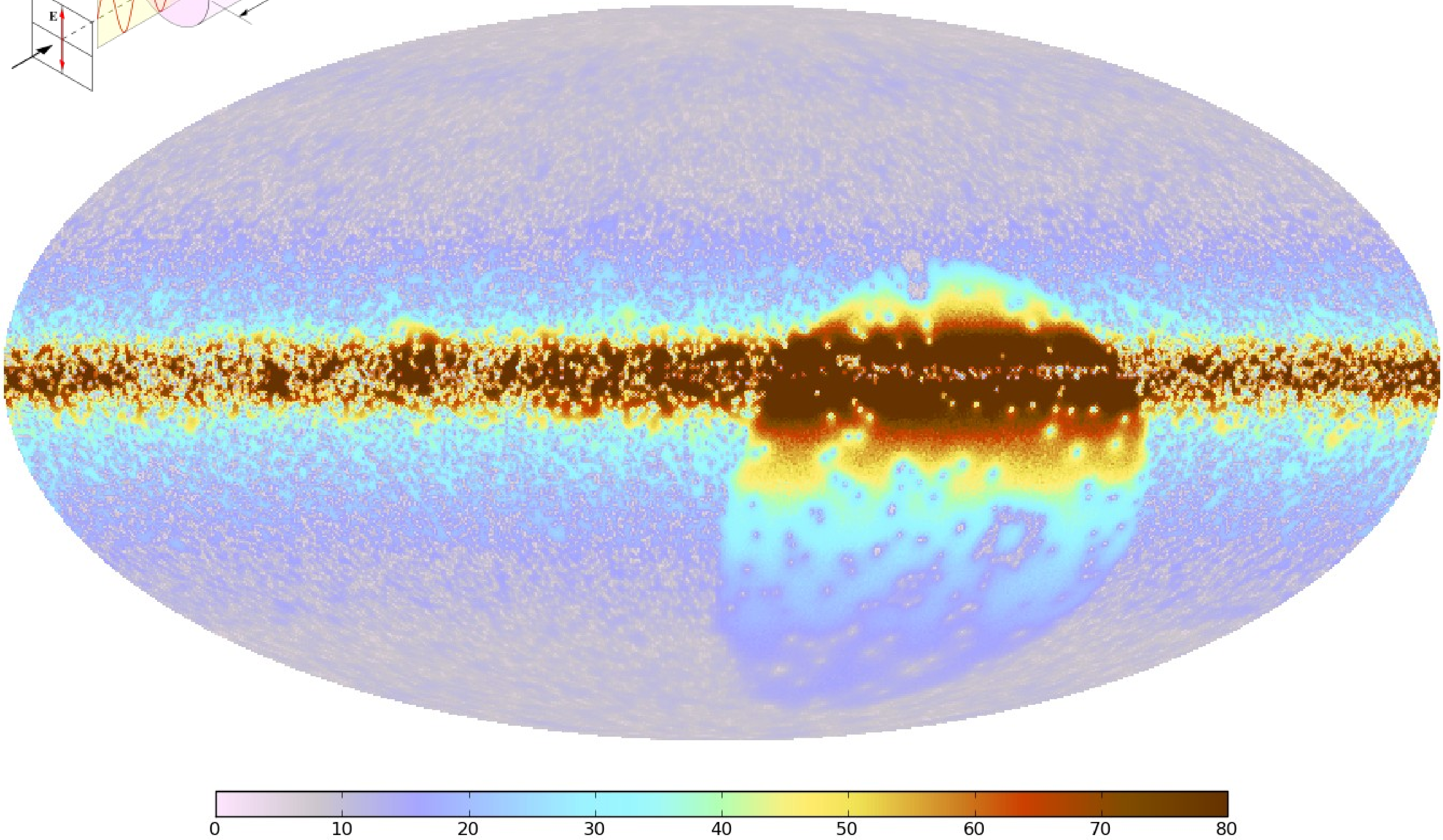

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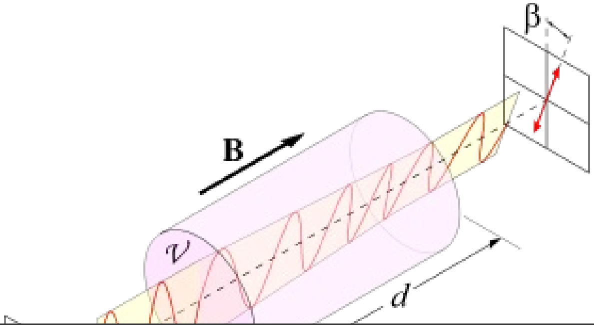

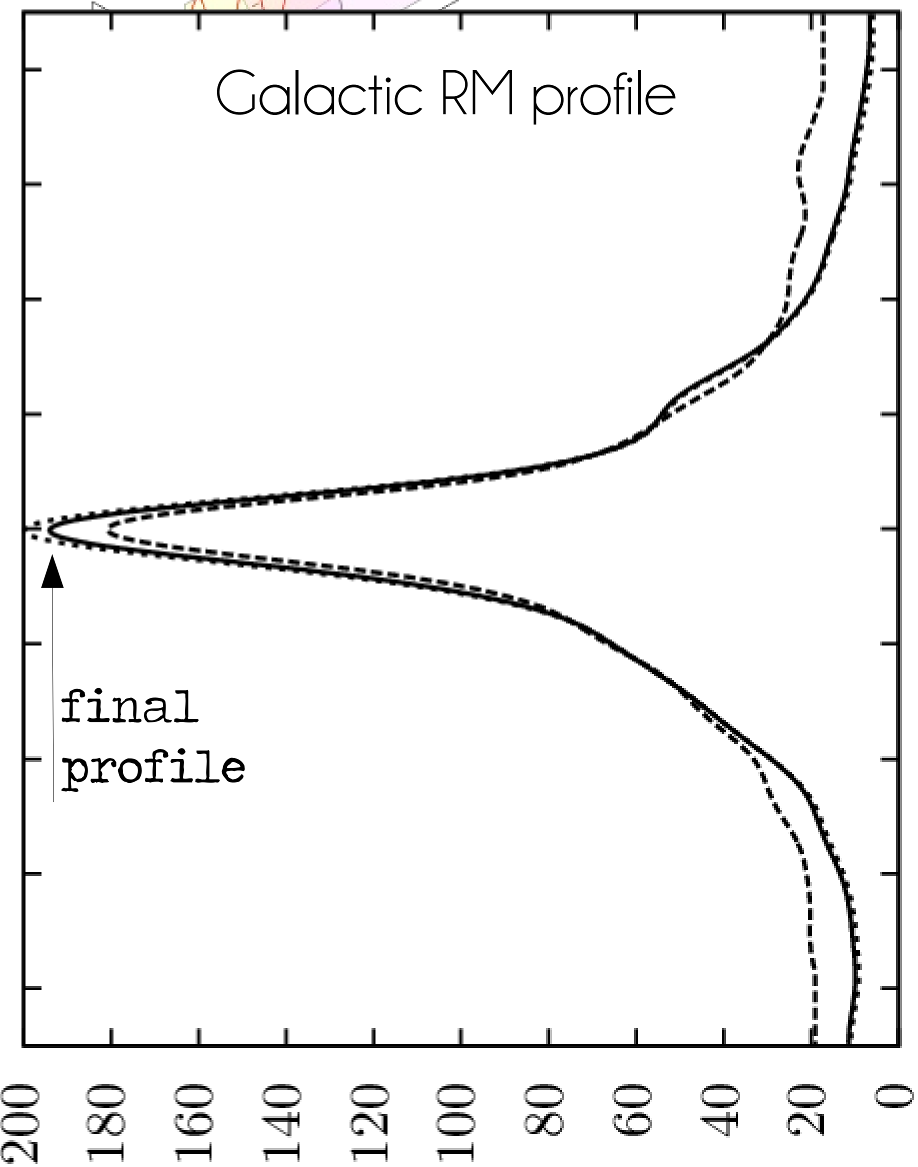

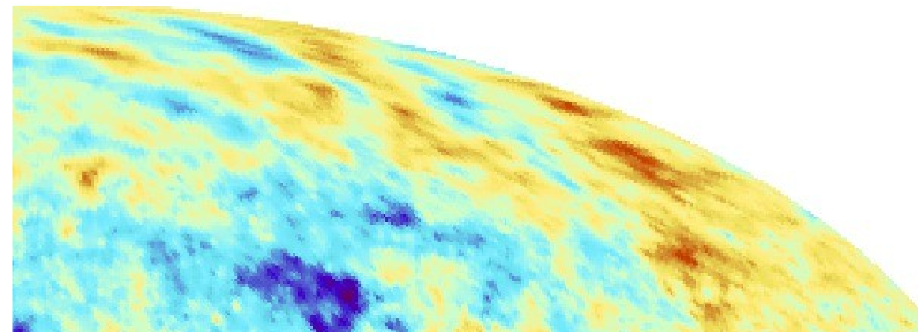

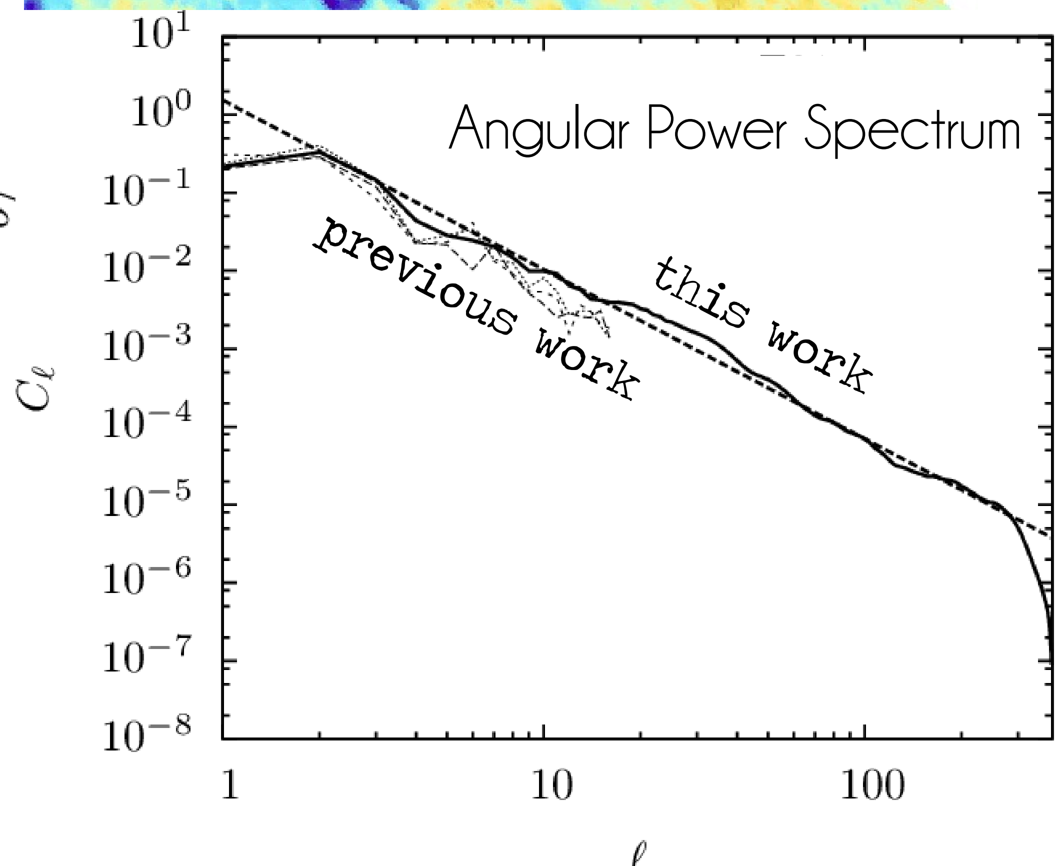

Crash-Course on IFT & NIFTy

Old-fashioned lecture series for serious cosmologists

Max Planck Institute for Astrophysics, Gaching
Wednesday July 10th & 11th:

Information field theory
Torsten Enßlin, lecture
Numerical Information Field Theory
Marco Selig, tutorial

http://www.mpa-garching.mpg.de/~komatsu/lectureseries/

Thank you!