Data Analysis for Continuum and Polarization

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Introduction

- Cover calibration, editing and imaging
- But not image analysis
- Step-by-step description
- Assume VLBI data

Theory: the Simple Picture

– Relation between data and images:

$$
\begin{aligned}
V_I(u,v) &= \int\int B_I(l,m) e^{-2\pi j(ul+vm)} dl\ dm \\
V_Q(u,v) &= \int\int B_Q(l,m) e^{-2\pi j(ul+vm)} dl\ dm \\
V_U(u,v) &= \int\int B_U(l,m) e^{-2\pi j(ul+vm)} dl\ dm \\
V_V(u,v) &= \int\int B_V(l,m) e^{-2\pi j(ul+vm)} dl\ dm
\end{aligned}
$$

– "Simply" have to invert equations

Theory: the Ugly Truth

For circularly polarized feeds measure in baseline ik using a piecewise approximation:

$$
\begin{aligned}
V_{RR}^{ikt\nu} &= g_{iR}g_{kR}^{*}b_{iR\nu}b_{kR\nu}^{*}e^{-2\pi j[(\tau_{iR}^{0}-\tau_{kR}^{0})(\nu-\nu_0)+(\dot{\tau}_{iR}-\dot{\tau}_{kR})(t-t_0)]} \\
&\quad \times\big[(V_I+V_V)e^{-j(\chi_i-\chi_k)} + (V_Q+jV_U)d_{Rk}^{*}e^{-j(\chi_i+\chi_k)} + \\
&\qquad (V_Q-jV_U)d_{Ri}e^{j(\chi_i+\chi_k)} + (V_I-V_V)d_{Ri}d_{Rk}^{*}e^{j(\chi_i-\chi_k)}\big] \\
V_{RL}^{ikt\nu} &= g_{iR}g_{kL}^{*}b_{iR\nu}b_{kL\nu}^{*}e^{-2\pi j[(\tau_{iL}^{0}-\tau_{kL}^{0})(\nu-\nu_0)+(\dot{\tau}_{iR}-\dot{\tau}_{kL})(t-t_0)]} \\
&\quad \times\big[-(V_I+V_V)d_{kL}^{*}e^{-j(\chi_i-\chi_k)} + (V_Q+jV_U)e^{-j(\chi_i+\chi_k)} - \\
&\qquad (V_Q-jV_U)d_{iR}d_{kL}^{*}e^{j(\chi_i+\chi_k)} + (V_I-V_V)d_{Ri}e^{j(\chi_i-\chi_k)}\big] \\
V_{LR}^{ikt\nu} &= g_{iL}g_{kR}^{*}b_{iL\nu}b_{kR\nu}^{*}e^{-2\pi j[(\tau_{iL}^{0}-\tau_{kR}^{0})(\nu-\nu_0)+(\dot{\tau}_{iL}-\dot{\tau}_{kR})(t-t_0)]} \\
&\quad \times\big[-(V_I+V_V)d_{iL}e^{-j(\chi_i-\chi_k)} - (V_Q+jV_U)d_{iL}e^{-j(\chi_i+\chi_k)} - \\
&\qquad (V_Q-jV_U)d_{kL}^{*}e^{j(\chi_i+\chi_k)} + (V_I-V_V)e^{j(\chi_i-\chi_k)}\big] \\
V_{LL}^{ikt\nu} &= g_{iL}g_{kL}^{*}b_{iL\nu}b_{kL\nu}^{*}e^{-2\pi j[(\tau_{iL}^{0}-\tau_{kL}^{0})(\nu-\nu_0)+(\dot{\tau}_{iL}-\dot{\tau}_{kL})(t-t_0)]} \\
&\quad \times\big[(V_I+V_V)e^{-j(\chi_i-\chi_k)} + (V_Q+jV_U)d_{Rk}^{*}e^{-j(\chi_i+\chi_k)} + \\
&\qquad (V_Q-jV_U)d_{Ri}e^{j(\chi_i+\chi_k)} + (V_I-V_V)d_{Ri}d_{Rk}^{*}e^{j(\chi_i-\chi_k)}\big]
\end{aligned}
$$

where

g_{iQ} is the complex gain of antenna i, polarization Q

$b_{iQ\nu}$ is the complex bandpass function of antenna i, polarization Q

τ_{iQ} is the group delay error of antenna i, polarization Q

$\dot{\tau}_{iQ}$ is the fringe rate error of antenna i, polarization Q

d_{iQ} is the polarization "leakage" term for antenna i, polarization Q

χ_i is the parallactic angle of antenna i

Step–by–step description

1) Correct phases for parallactic angle
   This is important for polarization data and removes the phase due to apparent rotation of the feed on the sky

$$g_{iR} = g_{iR}\, e^{-j\chi_i}$$

$$g_{iL} = g_{iL}\, e^{j\chi_i}$$

   This causes the source response to be fixed and the instrumental poln. to rotate with parallactic angle

2) Apply antenna gain, T_{sys}, weather to gains

Use external calibration information to get amplitude portion of gains g

Weather corrections needed for higher frequencies

Correlator measures ρ but want V:

$$|V| \;=\; |\rho||g_i||g_k| \text{ but for a point source } \rho \;=\; S(Jy)\,\sqrt{\frac{T^i_{sys}}{T^i_{ant}}}\sqrt{\frac{T^k_{sys}}{T^k_{ant}}}$$

$$T_{ant} \;=\; ant\_gain(K/Jy) * S(Jy)$$

$$|g_i| \;=\; \sqrt{\frac{S(Jy)T^i_{sys}}{ant\_gain^i(K/Jy) * S(Jy)}} \;=\; \sqrt{\frac{T^i_{sys}}{ant\_gain^i(K/Jy)}}$$

3) [optional] Apply phase cals

– Phase cals used to phase up multi frequency or multi polarization data

$$g_{ti} \;=\; g_{ti}\; e^{-j\phi_{ti}^{PC}}$$

– Tones injected early into signal path

– Corrects for instrumental phase and delay drifts

4) “Manual” Phase cal or correction

– Phase cals may be missing or inaccurate

– Can determine corrections using a “Fringe fit” on a section of strong calibrator data

– This will also refer all phase–like quantities to “reference” antenna

– Residual delay errors at time of calibration data will be corrected

5) Right-Left delay calibration [Polarization]

– There may systematic delay differences between RCP and LCP systems

– Must determine correction from cross polarized fringes
on strong polarized calibrator

$$\tau_{iL} \;=\; \tau_{iL} - \tau_{R-L}$$

– Should be constant (in absence of ionospheric Faraday rotation)

– $\Delta\dot{\tau}_{R-L}$ should be 0 (in absence of ionospheric Faraday rotation)

6) Editing

- This is actually a continious process
- Need to remove data during periods of equiptment malfunction or bad weather

7) Bandpass calibration

– Even for continuum data

– Bandpass mismatches introducing nonclosing errors

– These will not be removed by self calibration and will limit dynamic range

– Use data on source with strong fringes to determine $b_{i\nu}$ for each telescope/polarization

– Need enough SNR to get good determination in each channel

8) Fringe fit

– Determine $\arg(g_{iQt})$, τ_{iQt}, and $\dot{\tau}_{iQt}$ by dividing data into time segments where phase is linear in time and frequency

$$\phi \;=\; -2\pi[(\tau^0_{iR} - \tau^0_{kR})(\nu - \nu_0) + (\dot{\tau}_{iR} - \dot{\tau}_{kR})(t - t_0)]$$

– Fringe fitting a multi–dimensional self calibration

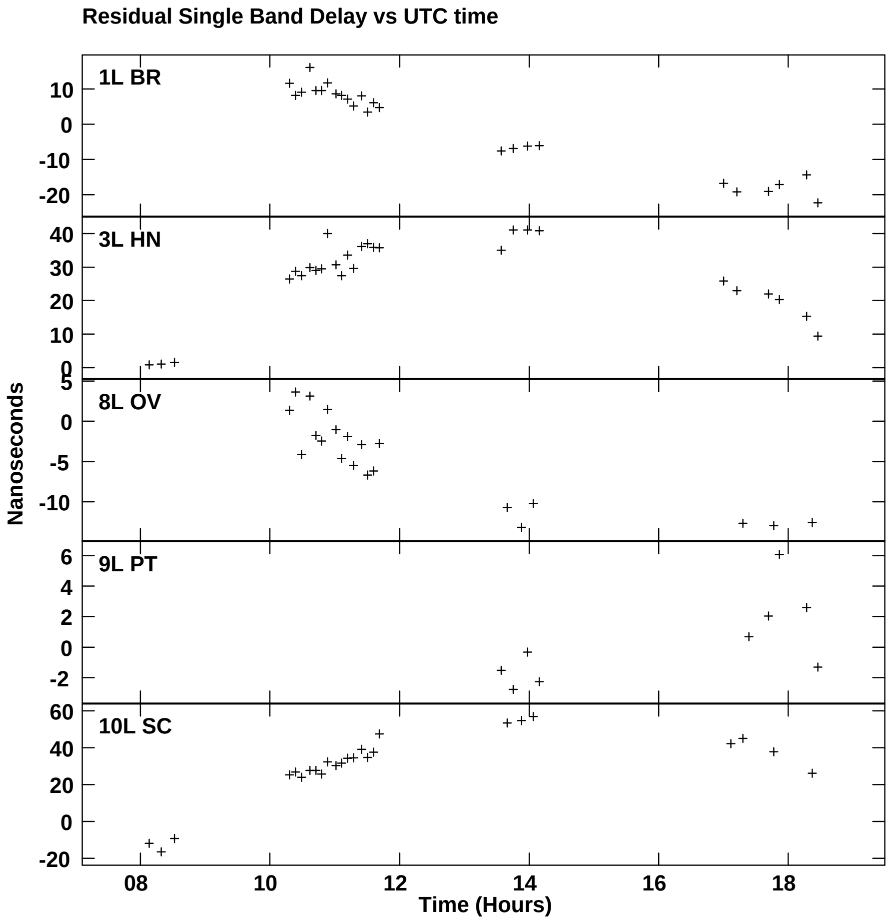
– Fourier transform V(t,ν) to V($\tau, \dot{\tau}$), position of peak gives $\tau, \dot{\tau}$

– If have model of source it can be used to remove structure phase before fringe fitting

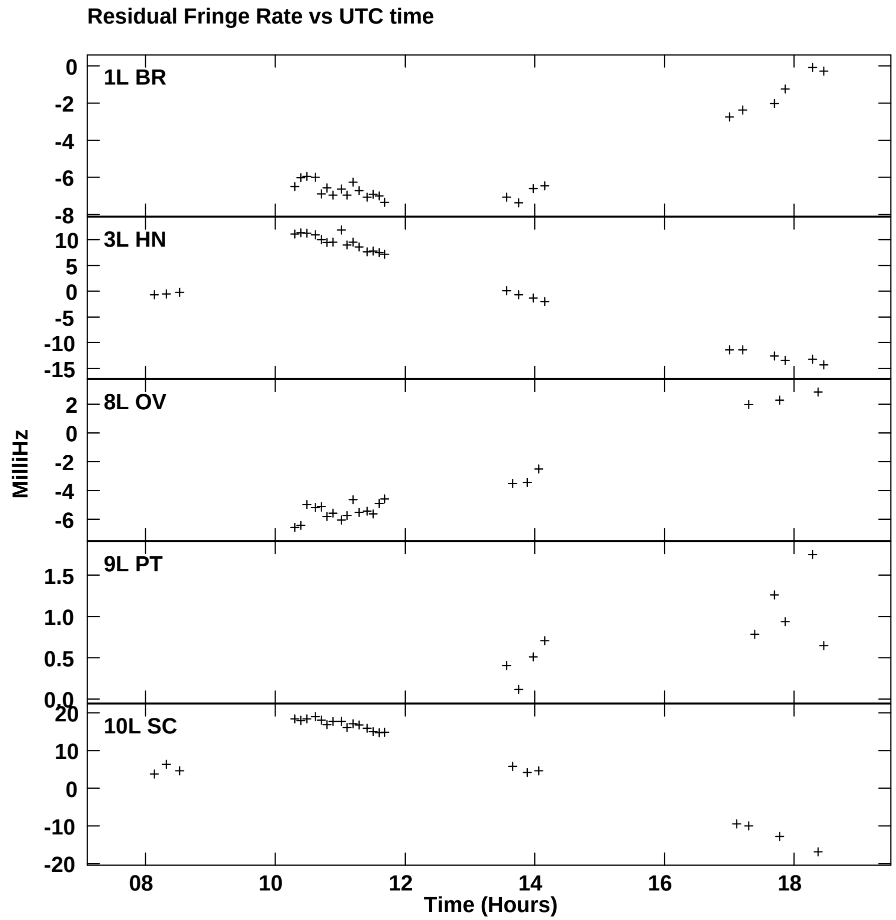
– If phase referencing only need to fringe fit calibrators

– However, the calibrators should be completely self calibrated before final fringe fit

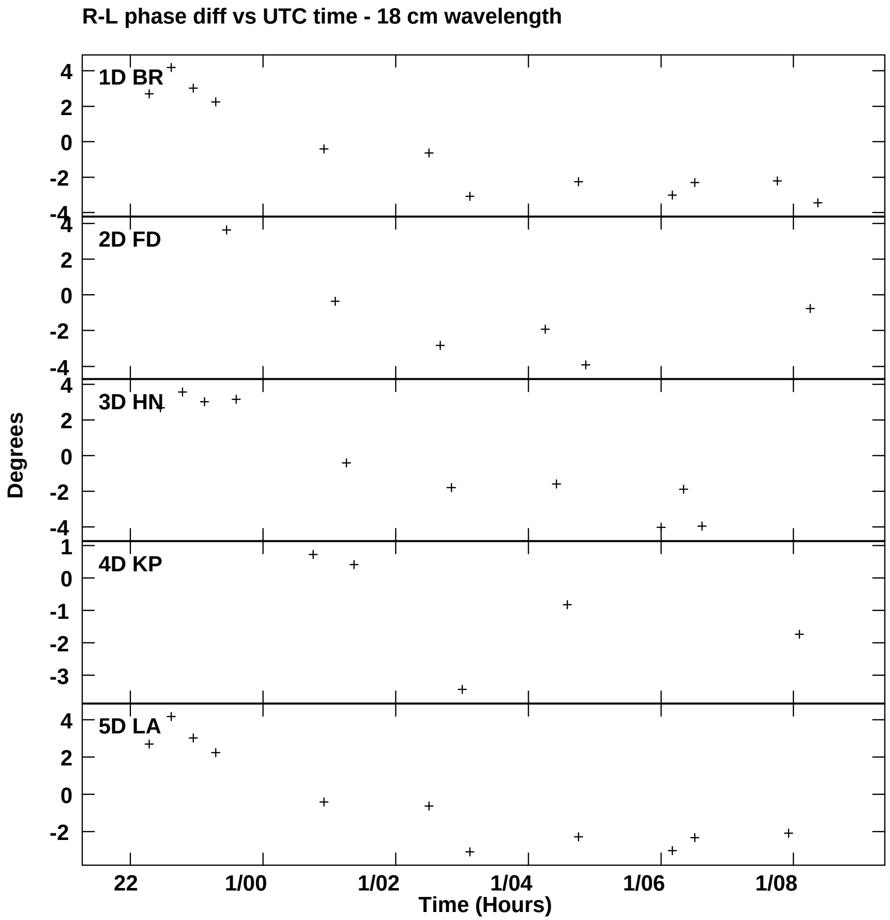
– Check right-left coherence for polarization measurements



Residual delays determined from a fringe fit



Residual rates determined from a fringe fit



Residual right-left phase differences at 18 cm showing slight ionospheric Faraday rotation

9) Average in time and frequency

- Not really necessary but dramatically reduces the volume of data
- After applying fringe fit calibration average in time and frequency
- Averaging should consider the desired field of view and subsequent self calibration

10) Self calibration

– Calibrators – if not already done

– Target sources if not phase referencing

– If know calibrator total flux densities the overall amplitude calibration can be improved

11) Instrumental polarization [Polarization]

- May need for high dynamic range with hetergeneous array
- Generally need to solve for both source and instrumental polarization
  - Source polarization rotates with sum of parallactic angles
  - Instrumental terms rotate with difference
  - Precalibration ignoring second and higher order terms:

$$
\begin{aligned}
V_{RL}^{ik} &= gain \times \left[V_I(d_{iR}e^{j(\chi_i-\chi_k)} + d_{kL}^{*}e^{-j(\chi_i-\chi_k)}) - (V_Q + jV_U)e^{-j(\chi_i+\chi_k)}\right] \\
V_{LR}^{ik} &= gain \times \left[\underbrace{V_I(d_{iL}e^{-j(\chi_i-\chi_k)} + d_{kR}^{*}e^{j(\chi_i-\chi_k)})}_{\text{Instrumental}} - \underbrace{(V_Q - jV_U)e^{j(\chi_i+\chi_k)}}_{\text{Source}}\right]
\end{aligned}
$$

- Need observations of one or more calibrators over a range of parallactic angle
- Unresolved or unpolarized source works best

12) Image calibrators for R-L phase [polarization]

- Polarization angle unconstrained by total intensity calibration
- An offset in polarization angle is equivalent to right-left phase offset
- Need to know actual integrated polarization angle of a calibrator, or of a given component

$$\phi_{R-L}^{true} \;=\; 2.0 \times true\_polarization\_angle$$

- Can determine apparent R-L phase from the sums of the Q and U components from CLEAN

$$\phi_{R-L}^{apparent} \;=\; Tan^{-1}(\frac{\Sigma U}{\Sigma Q})$$

- The polarization angle calibration is completed using the gains:

$$g_{iLt} \;=\; g_{iLt}\; e^{-j(\phi_{R-L}^{true} - \phi_{R-L}^{apparent})}$$

13) Image/deconvolve

- Convert from (RR,RL,LR,LL) to (I,Q,U,V) applying calibration
- Image/deconvolve Stokes' I, Q, U and V as needed

14) Astrophysical interpretation of image

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Things that go Bump in the Night

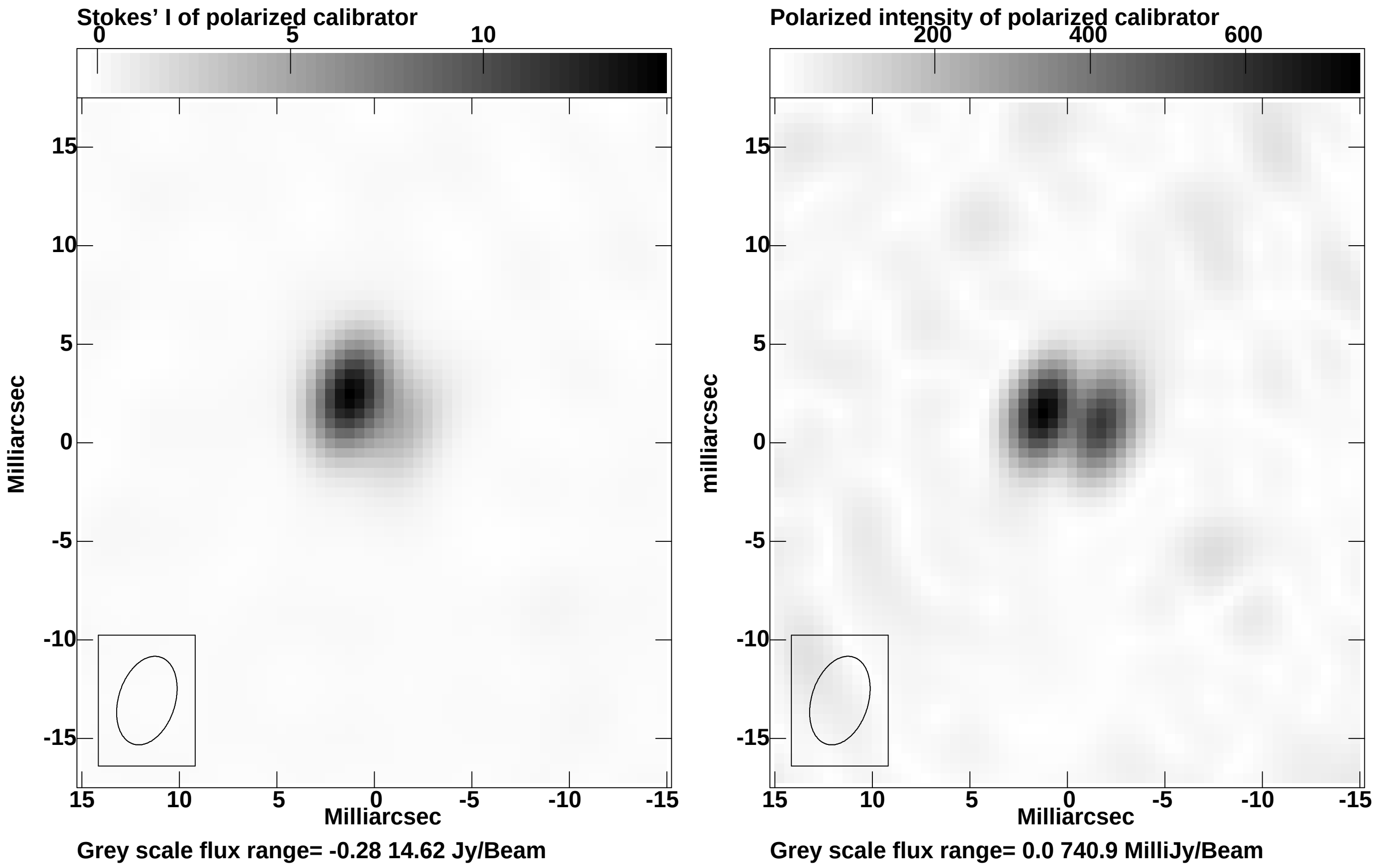
- Ionospheric Faraday rotation
- Resolved, polarized instrumental polarization calibrators

Ionospheric Faraday rotation

- Longer than 10 cm wavelength the ionosphere can cause Faraday rotation
- If uncorrected will corrupt Stokes' Q and U images
- Effect varies with time and observing geometry
- Can use GPS measurements to correct
- Or "selfcalibrate" (but lose absolute polarization angle)

Resolved, polarized instrumental polarization calibrators

- Problem determining instrumental polarization
- "Similarity" approximation: Q and U are scaled versions of I
- Breaks down very quickly
- Can divide source into components with scales Q, U



Total and polarized intensity of resolved, polarized calibrator