Radio and Optical Interferometry

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Introduction

- Cover radio and optical interferometry
- Basic physics the same but technology and details differ
- Cover basic concepts and techniques

The Quantum Limit

– Heisenberg Uncertainty Principle:

$$\Delta E \ \times \ \Delta t \ = \ \frac{h}{2\pi}$$

Knowledge of time (phase) cost knowledge of energy

– In terms of phase and number of photons:

$$\Delta\phi \ \times \ \Delta n \ = \ 1$$

– Measuring phase adds uncertainty to the number of photons received

– This effectively adds noise (system temperature):

$$T_{sys}^{QM} = \frac{h\nu}{k\Delta\phi}$$

at 1.4 GHz, $T_{sys}^{QM} = 0.07$ K
at 230 GHz, $T_{sys}^{QM} = 11$ K
at 900 GHz, $T_{sys}^{QM} = 43$ K
at 500 THz (optical), $T_{sys}^{QM} = 30{,}000$ K

Basic Interferometers

- Many ways to construct an interferometer
  - Adding interferometer

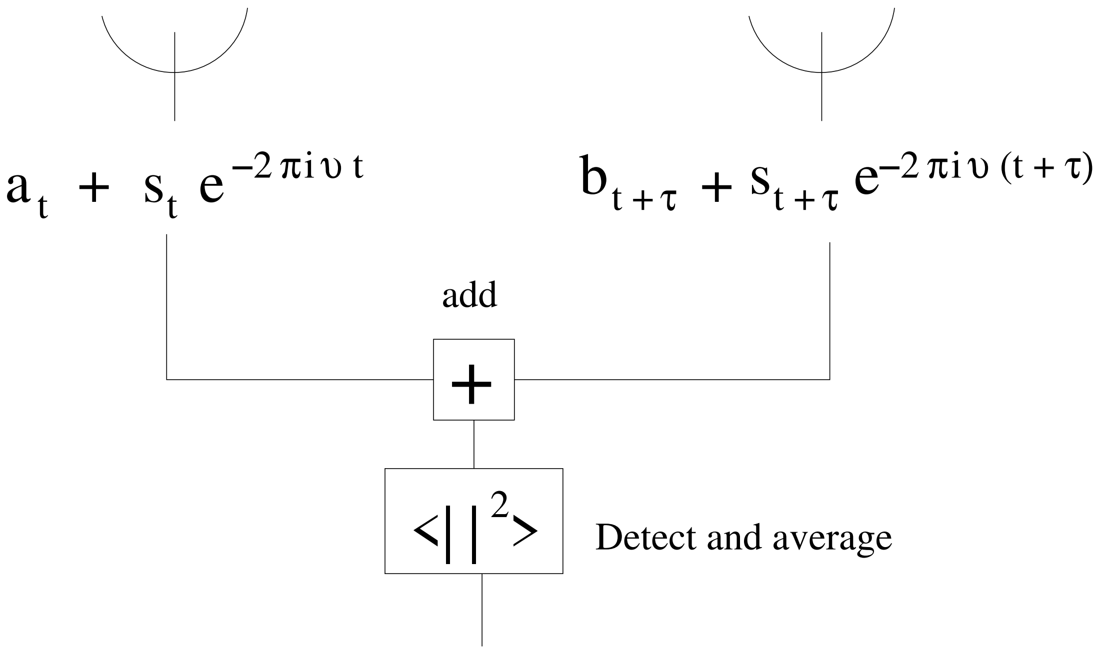
    output $= < a^2 > + < b^2 > + < 2S^2(1 + cos[2\pi\nu\tau]) >$

    where τ is opd (delay difference), ν = frequency,
  - Correlation interferometer

    output $= < 2S^2 cos[2\pi\nu\tau] >$
- Heterodyne (radio) interferometer
  - Measure multiple lags simultaneously
  - More baselines cost $ not sensitivity

Adding Interferometer

(monochromatic)

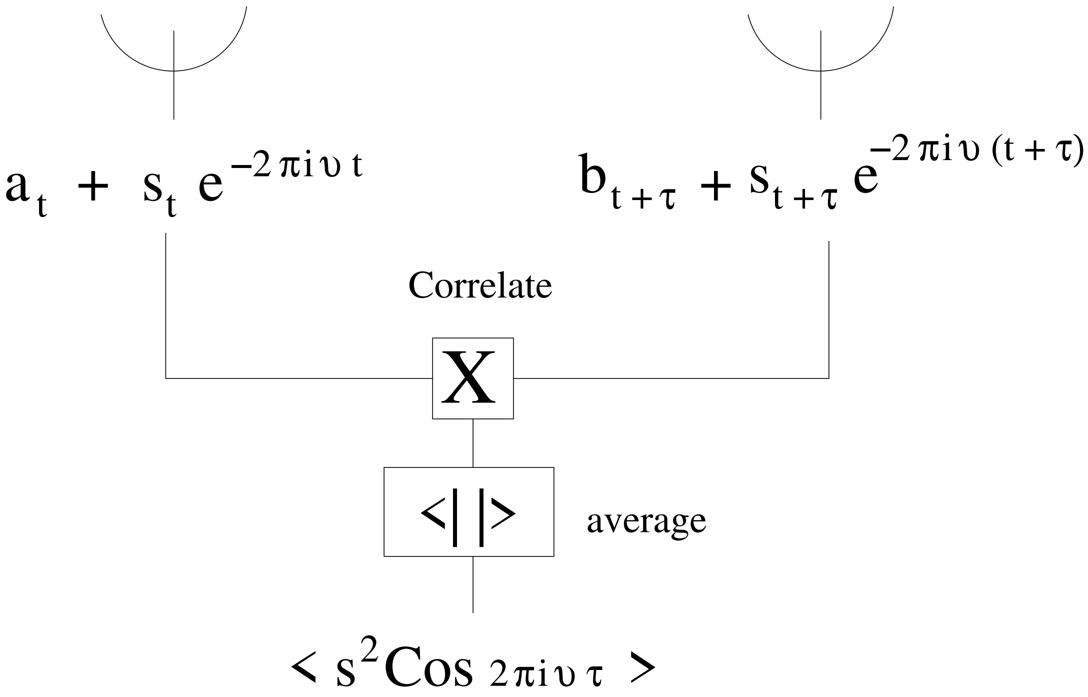


$$\langle a^2\rangle + \langle b^2\rangle + \langle 2s^2(1 + \mathrm{Cos}\, 2\pi i \upsilon \tau)\rangle$$

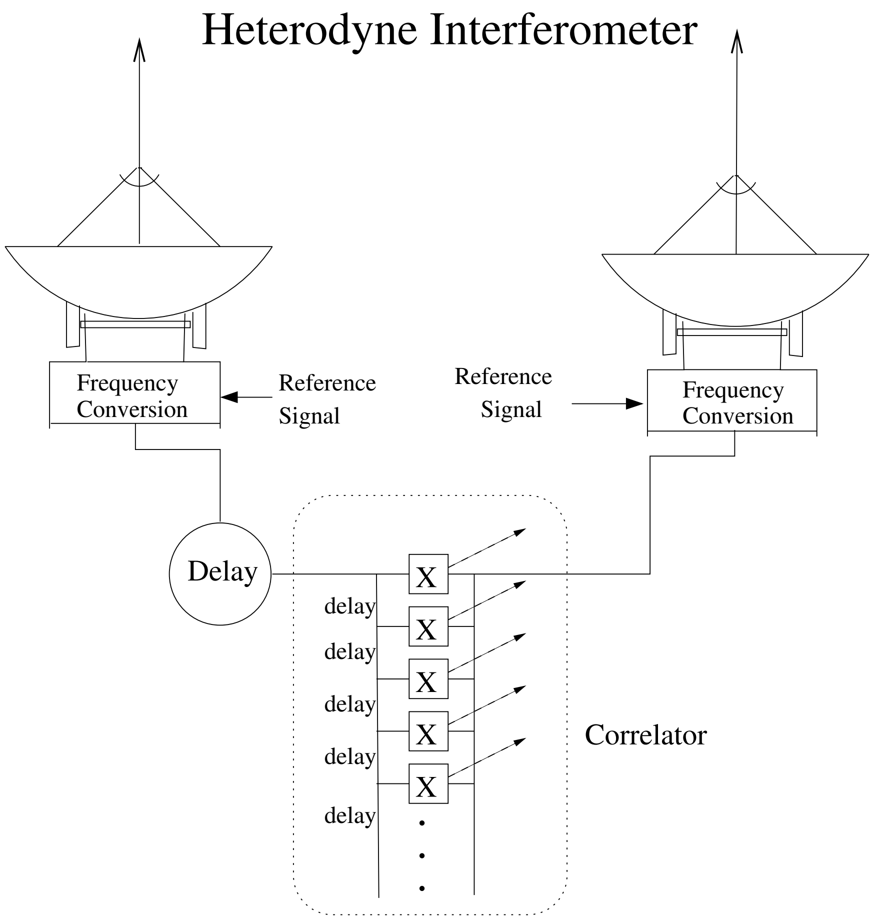
Simple quasimonochromatic adding interferometer

Correlation Interferometer

(monochromatic)



Simple quasimonochromatic multiplying (correlation) interferometer



Multilag heterodyne interferometer

Finite Bandwidth and Delay Compensation

- For a finite bandwidth, β the interferometer response becomes

  output $= < a^2 > + < b^2 > + < 2S^2(1 + cos[2\pi\nu\tau])sinc(\beta\tau) >$

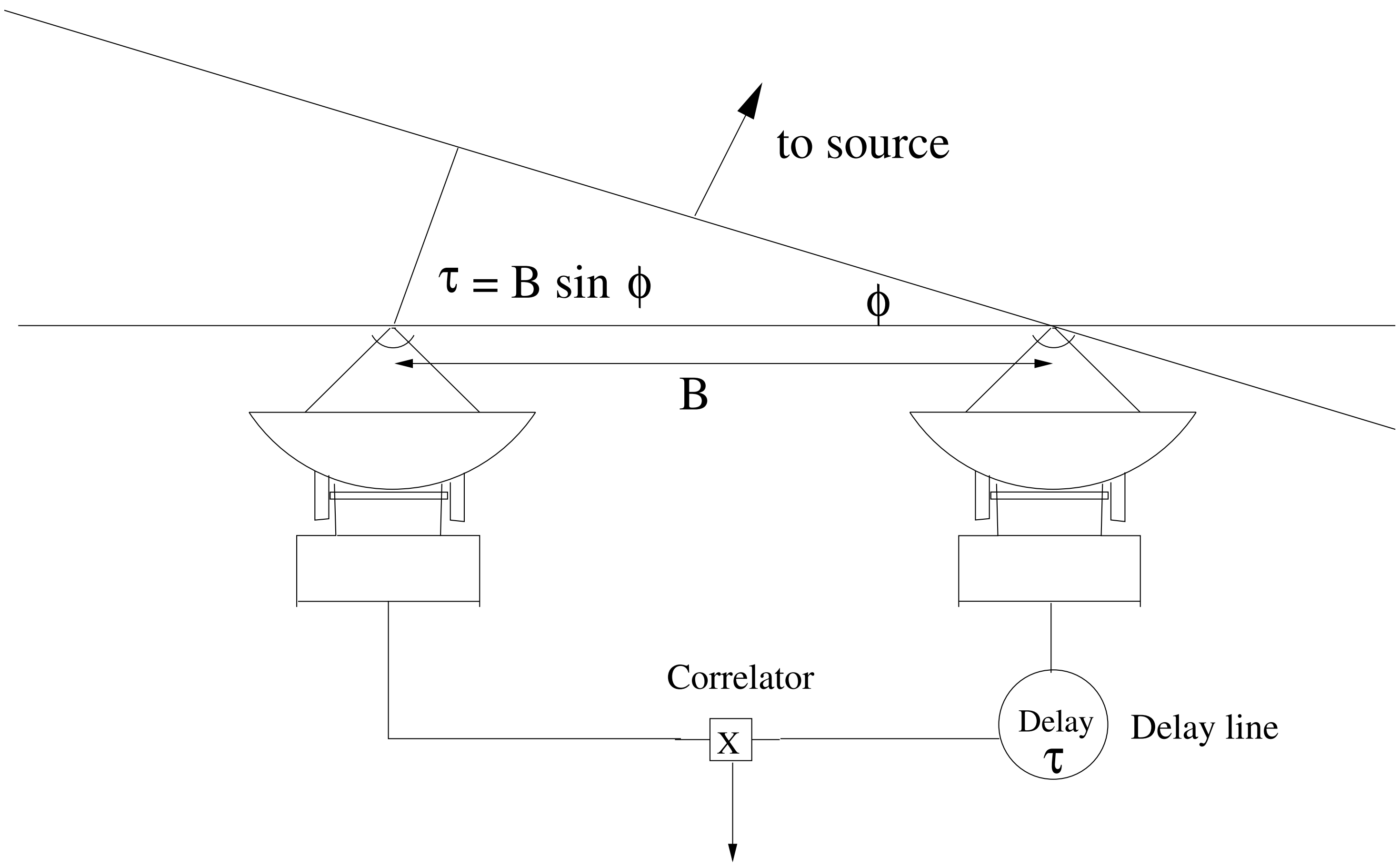
  where $sinc(x) = \frac{sin(\pi x)}{\pi x}$

  for an adding interferometer or for a correlation interferometer:

  output $= < 2S^2(1 + cos[2\pi\nu\tau])sinc(\beta\tau) >$

  ∘

- $sinc(\beta\tau)$ is 1 near $\beta\tau = 0$ and decreases away
- To avoid loss of coherence must equalize the delays
- Can measure multiple delays if not known well
  - Heterodyne (radio) interferometer can measure simultaneously
  - Direct (optical) may modulate delay



Interferometer with delay compensation

Time lag – Sky Frequency Relation

Lag correlation function and sky frequency related by Fourier transform:

$$V(\nu) \;=\; c \int S(\tau) e^{-2\pi i \nu \tau} \; d\tau$$

or

$$S(\tau) \;=\; \frac{1}{c} \int V(\nu) e^{2\pi i \nu \tau} \; d\nu$$

Can measure spectrum from lag function,
or lag function from spectrum

Spectroscopy

- With heterodyne system can get arbitrary resolution with sufficient lags in correlator
- With direct system better to disperse before detection

Source Structure

– The response to an extended source with brightness distribution $I(l, m)$ is:

$$V(u, v, w) = \int\int A(l, m) I(l, m) e^{-2\pi i[ul+vm+w(\sqrt{1-l^2-m^2}-1)]} \frac{dl\ dm}{\sqrt{1 - l^2 - m^2}}$$

where A is the primary telescope pattern, (l, m) are coordinates on the sky, (u, v, w) is the baseline vector in the coordinate frame of the source

– If the array is 2D or the source is sufficiently small this can be approximated by a 2D FFT

$$V(u, v) = \int\int A(l, m) I(l, m) e^{-2\pi i[ul+vm]} \frac{dl\ dm}{\sqrt{1 - l^2 - m^2}}$$

which can be inverted:

$$\frac{A(l, m) I(l, m)}{\sqrt{1 - l^2 - m^2}} = \int\int V(u, v) e^{2\pi i[ul+vm]} du\ dv$$

Aperture Sampling and Imaging

- Each baseline traces an elipse in uv plane as earth rotates
- Only sample some (uv); $V(u, v)$ not sampled everywhere
- Cannot invert equation for $I(l, m)$
- Introduce Sampling function S(u,v):

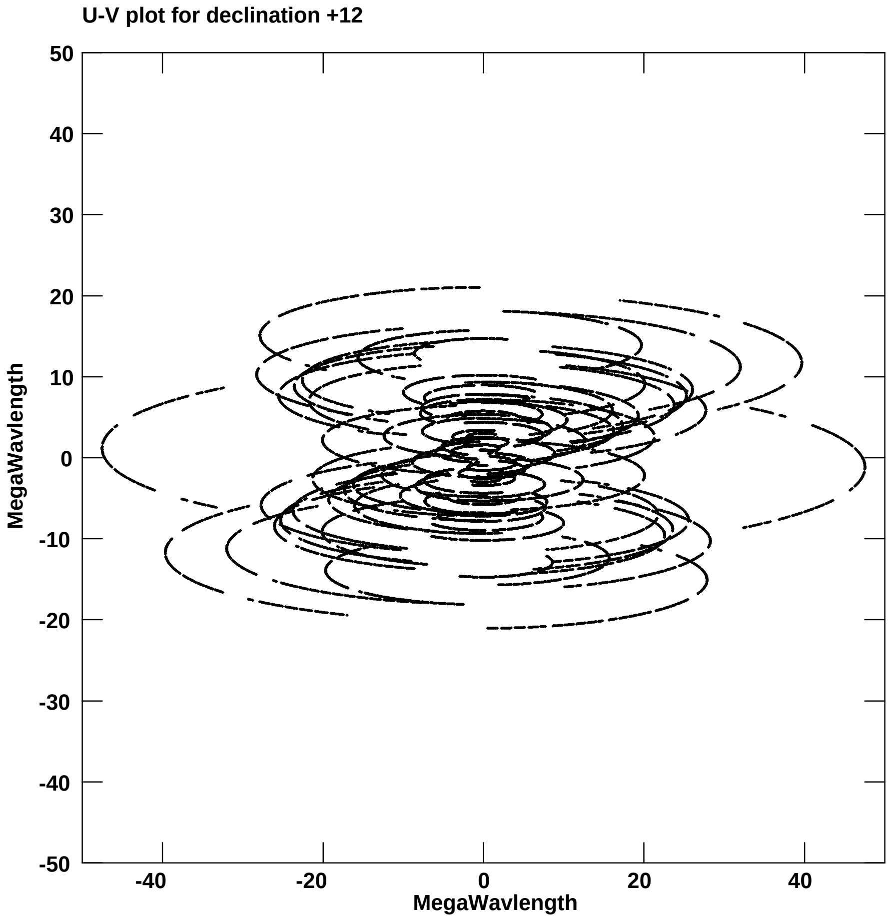
  $S(u, v) = 1.0$ where sampled otherwise 0.0
- Define sampled visibility V'

$$V'(u, v) \;=\; V(u, v)\; S(u, v)$$

  which is defined everywhere
- Can Fourier transform $V'(u, v)$ and get $D(l, m)$

$$D(l, m) \;=\; \int\!\!\int V'(u, v) e^{2\pi i[ul+vm]} du\; dv$$

- $D(l, m)$ called "Dirty" image



Typical UV coverage for VLBA, 10 antennas, 45 baselines

Deconvolution

– By the Fourier convolution theorem, $D(l,m)$ is the convolution of the true image with the Fourier transform of the Sampling function $B(l,m)$

$$D(l,m) \;=\; A(l,m)I(l,m) \;\star\; B(l,m)$$

– (From here on assume $A(l,m) = 1$ and ignore)

$$D(l,m) \;=\; I(l,m) \;\star\; B(l,m)$$

– Need to deconvolve to recover $I(l,m)$

– Must be nonlinear deconvolution ($S(u,v)$ is full of zeroes)

– CLEAN and Maximum Entropy most popular

CLEAN

- CLEAN algorithm due to Högbom
  1) Find peak brightness pixel in image
  2) Subtract dirty beam times fraction of peak from image
  3) Repeat until converged
  4) Convolve "CLEAN" components with nice beam and add to residuals
- Decomposed image into delta functions
- Very robust it always converges
- CLEAN components are a model of the image

Atmospheric (and other) Delays

- Atmospheric delay known well
- Geometric and related delays may be uncertain as well
- Always have moving delay (phase) screen across array
- Which part of atmosphere depends on wavelength
  - meter wavelength - ionosphere
  - cm wavelength - water vapor in lower troposphere
  - Optical/IR - density fluctuation in shear layers in neutral component
- Requires delay and/or phase calibration

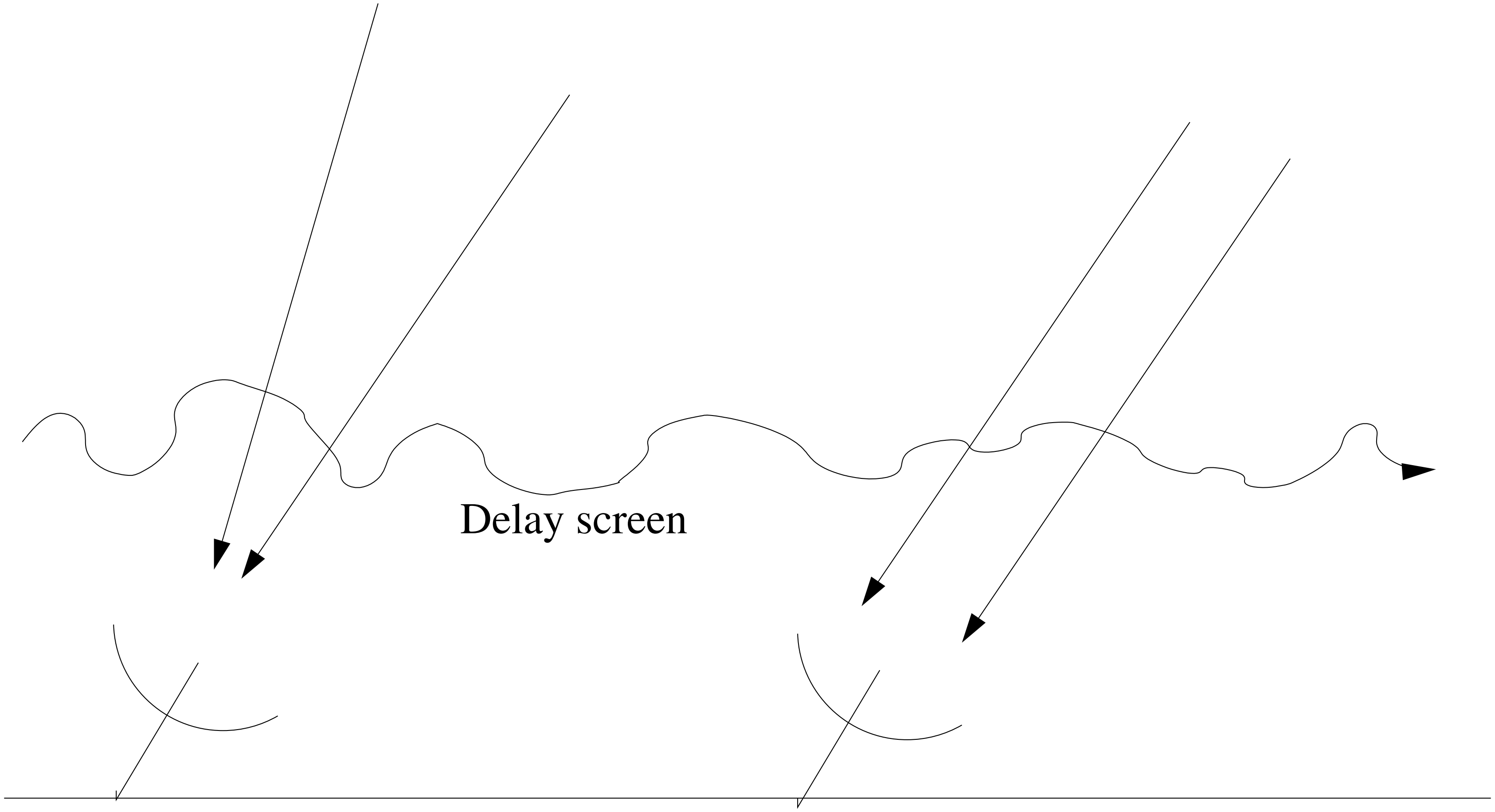


Illustration of delay screen across the array

Phase Calibration

- Most phase errors depend on the telescope
- Observation of a nearby reference source can determine phase corruption for target
  - Switch source rapidly in radio
  - Observe simultaneously in Optical/IR
- Self calibration in radio if source strong enough
- Bispectrum in optical/IR (related to self cal)

Phase corruption by telescope

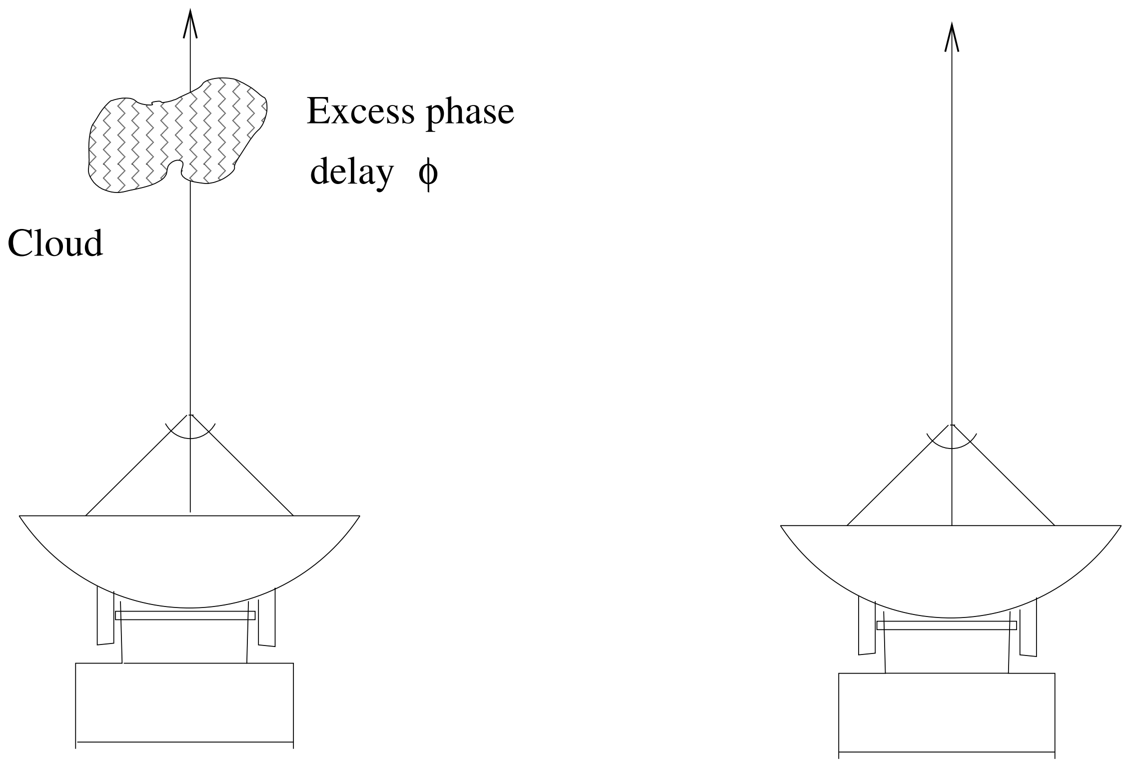


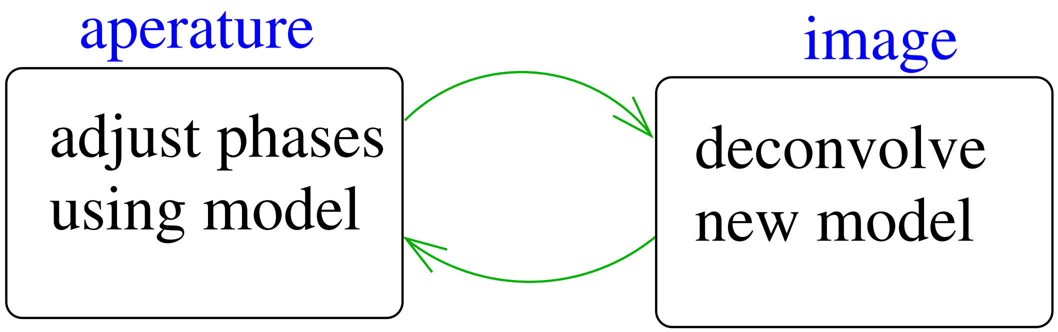
Illustration of cloud afecting the phase of one antenna

Self Calibration

- Phases depend on telescope and have more baselines than telescopes
- Use redundant information to solve for phase and source structure

  For n telescopes need n-1 phases

  but have n×(n-1)/2 phases
- Use CLEAN (or other) model to recalibrate data and iterate



Optical interferemetry

- Only Adding (Direct) interferometers
- Many current interferometers only measure amplitude
- Delay lines mirrors on moving carts
- Mechanical stability VERY important
- Need tip–tilt correction and adaptive optics for large telescopes

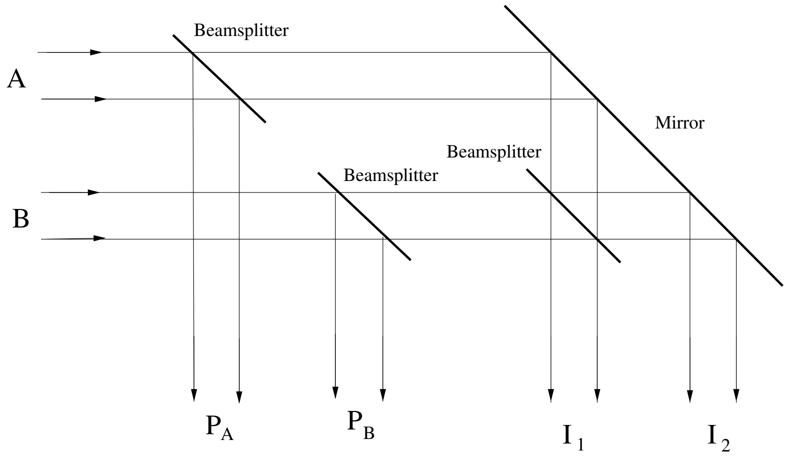
Tourist Guide to Interferometry Jargon

Optical/IR speak	Radio speak
Optical path difference (OPD)	Delay, lag
Differential piston	Delay residual
Beam combiner	Correlator
Strehl ratio	Antenna gain
Background level	System temperature
Fringe tracking	Phase referencing
Telescope	Antenna
Detector	Feed
Point spread function (PSF)	Dirty (or CLEAN) beam
Magnitudes	log (flux density)
Obscure band designations	Confusing band designations

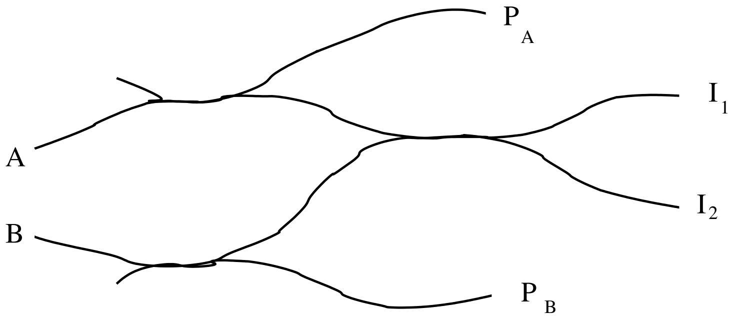
Beam Combination

Beam combination may be in vacuum (or air) or single mode fibers

Vacuum Beam Combiner

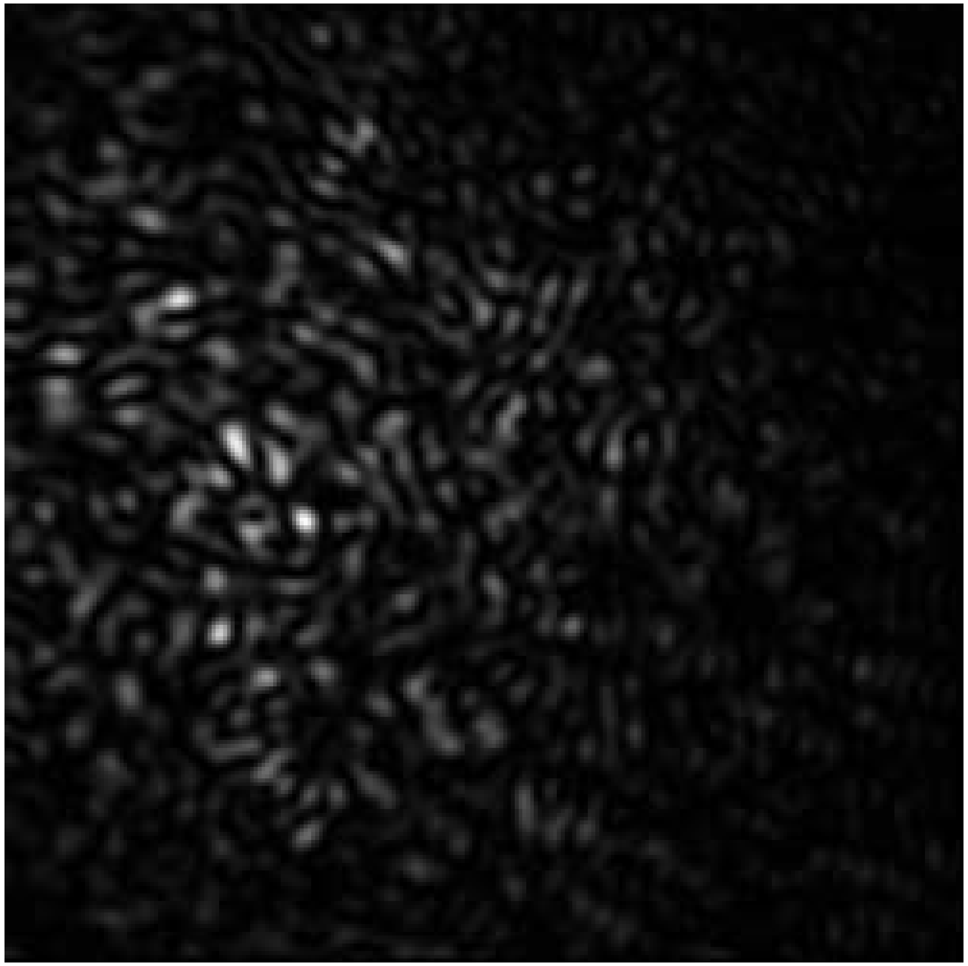


Fiber Beam Combiner



Atmospheric Phase

- variable correlation of aperture – > variable Strehl ratio
- Coherence size in aperature (r_0) varies from few cm in blue to few m in IR
    - “Seeing” breaks image into speckles
- Correct with Tip-tilt or adaptive optics
- Filter single speckle with pinhole or single mode fiber



Simulated speckle pattern r_0=0.2m, telescope = 8m

References

– Thompson, Moran, Swenson, **Interferometry and Synthesis in Radio Astronomy**, 2001, John Wiley and Sons, New York

– **Synthesis Imaging in Radio Astronomy II**, 1999, Taylor, Carilli, Perley, eds. ASP.