Image processing and enhancement (amplitude Calibration and self-calibration)
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Gathering ideas
1. principles of “radio observations”
2. basic relations of calibration

Handling real “noise” [data]
3. structure of your visibility
4. Inspection of raw data
5. Applying a-priori information
6. (double) checking results
7. calibration transfer
8. Fourier inversion

Improving images
9. Principles and options of self-calibration
10. Self-calibration in practice
11. (double) checking self-cal
12. when to stop iterating....(when the is game over)
Wandering among real facts and assumptions:

**SUMMARY**

- Far Far Away signal comes to the Earth “undisturbed” -
  
  [... not true, i.e. Faraday Rotation]

- Crossing the atmosphere introduces some disturbance

- The receiving system (mirror + feed + electronics) introduces some additional “noise” and disturbance

- All the disturbances can be assigned to each element and combine linearly

- All the disturbances vary smoothly with time and can be appropriately monitored and removed (CALIBRATION)

- The process of CALIBRATION is aimed to transform each measurement (in its own units) into a “sample of the Visibility Function”
Each telescope samples the incoming (rapidly varying) E field

The combination (interferometer) has the resolution corresponding to the separation between elements, but a much worse PSF

Multi-element interferometer: each pair $A_iA_j$ measures the visibility function together with a substantial noise contribution, better PSF but still unsatisfactory

**IMAGING** is necessary to derive an image of the radio sky at a given frequency from the sampled Visibility Function (Fourier Inversion & Cleaning)

The interferometric image is still difficult to interpret since lots of details may be hidden by sidelobes, **CLEAN** is necessary

Second order errors (residual short timescale phase [ & gain] variation) create artifacts. They can be corrected/removed by **SELF-CALIBRATION**

The final image (sample, measure, spectrum...) is a fair representation of the extra-terrestrial signal (brightness distribution, spectrum, ...)
Hardware or control software occasionally fails or behaves unpredictably [e.g. power failure, computer problems, ...]

Scheduling/observation errors sometimes occur (e.g., wrong/unknown source positions)

Atmospheric conditions not ideal (not just bad weather) and may vary during the experiment

RFI and power glitches .... and more

all these events corrupt the incoming signal, which needs to be handled before the “final image” is produced


Gathering Ideas: 1. principles of “radio observations”

**Interferometer**

Ideally we wish to obtain the visibility function, which we intend to invert to obtain an image of the sky brightness distribution:

\[
V(u, v) = \int B_v(\theta, \phi) \, e^{-2\pi i (u\theta + v\phi)} \, d\theta \, d\phi
\]

In practice, we correlate the electric field (voltage) samples taken at pairs of telescopes (baselines i-j), sampling a single spacing at a time:

\[
V_{ij}^{obs}(t) = \langle x_i(t) e^{i \phi_i} \cdot x_j^*(t) e^{i \phi_j} \rangle_t
\]

Single radio telescopes collect the signal \(x_i(t)\) and provide it to the correlator.
The net signal delivered by antenna \( i \), \( x_i(t) \), is a combination of the desired signal, \( s_i(\theta, \phi, t) \), corrupted by a factor \( C_i(\theta, \phi, t) \) and integrated over the sky, and noise, \( n_i(t) \) which is unwanted power added by various contributors:

\[
x_i(t) = \int C_i(\theta, \phi, t) s_i(\theta, \phi, t) \ d\theta \ d\phi = s_i'(t) + n_i(t)
\]

\( C_i(\theta, \phi, t) \) is an antenna-based complex number
accounts for many effects which have to calibrate

In case the effects contained in the \( C_i(\theta, \phi, t) \) term corrupt unrecoverably the signal (pointing, receiver saturation...), the resulting data must be edited out

In general at any given station \( n_i(t) \) largely outshines \( s_i(\theta, \phi, t) \) ..... but

the correlation of two signals from different antennas helps a lot!
The correlation of two realistic signals from different antennas:

\[
\langle x_i(t) \cdot x_j^*(t) \rangle = \langle (s'_i(t) + n_i(t)) \cdot (s'_j(t) + n_j(t))^* \rangle \\
= \langle (s'_i \cdot s'_j^*) \rangle + \langle (s'_i \cdot n_j^*) \rangle + \langle (n_i \cdot s'_j^*) \rangle + \langle (n_i \cdot n_j^*) \rangle \\
\approx \langle (s'_i \cdot s'_j^*) \rangle \\
\langle x_i(t) \cdot x_j^*(t) \rangle = \int C_i(\theta, \phi, t) C_j^*(\theta, \phi, t) s_i(\theta, \phi, t) s_j^*(\theta, \phi, t) d\theta d\phi
\]

Noise doesn’t correlate: correlation isolates desired signal!

In integral, only \( s_i(\theta, \phi, t) \) from the same direction correlate, so order of integration and signal product can be exchanged and terms re-ordered
The FOV is limited (further: are there any known strong confusing sources?)

- primary beam \[ \theta_{\text{FOV}} \approx \frac{1}{2} \theta_{\text{antenna}}^{\text{HPBW}} \]

- bandwidth smearing \[ \theta_{\text{FOV}}^{\text{BW}} \approx \frac{\nu_{\text{obs}}}{\Delta \nu} \theta_{\text{HPBW}}^{\text{synth}} \]

- time smearing \[ \theta_{\text{FOV}}^{\text{TA}} \approx \frac{\theta_{\text{HPBW}}^{\text{synth}}}{\omega_{\text{earth}} \tau_{\text{a}}} \]

The bandwidth and the time smearing need to be arranged at the time of scheduling: \( \tau_{\text{a}} \) and \( \Delta \nu \) have to be properly chosen.
Gathering Ideas: 1. principles of “radio observations”

Visibility function written in a more complete form:

\[ V(u, v, w, \nu) = V_0(u, v, w, \nu) e^{-i\phi(u, v, w, \nu)} \]

Telescopes generally sample RCP and LCP independently (eventually X and Y)
The R and L “systems” must be treated independently and each measures

\[ V_R(u, v, w, \nu) = V_{R,0}(u, v, w, \nu) e^{-i\phi_R(u, v, w, \nu)} \]
\[ V_L(u, v, w, \nu) = V_{L,0}(u, v, w, \nu) e^{-i\phi_L(u, v, w, \nu)} \]

N.B.
The correlator may produce “parallel hand” correlations (RR, LL) sensitive to total intensity \[ I = (RR + LL)/2 \] and circular polarization \[ V = (RR - LL)/2 \] and “cross hand” products (RL, LR) sensitive to linear polarization
\[ U = RL + LR; \quad Q = i(LR - RL) \]; if \( V=0 \) then \( RR = LL \)
Monochromatic radiation interpreted as the superposition of two waves with opposite polarization: RCP and LCP (indeed single feed + OMT) [crossed dipoles]

Telescopes generally sample RCP and LCP independently (eventually X and Y). The R and L “systems” must be treated independently.

\[
E_{i(R,L)}(t) = E_{o(R,L)} \sin[\omega t + \phi_{(R,L)}] + n_{i(R,L)}(t)
\]

\[
E_{j(R,L)}(t + \tau_g) = E_{o(R,L)} \sin[\omega (t + \tau_g) + \phi_{(R,L)}] + n_{j(R,L)}(t)
\]

Correlation of these two, for a given polarization, on the ij baseline (terms with noise are 0 over integration time):

\[
E_i E_j \approx E_{o}^2 \cos(\omega \tau_g) = E_{o}^2 \cos(2\pi \nu \frac{D}{c} \sin \theta) = E_{o}^2 \cos(2\pi \frac{D}{\lambda} \sin \theta)
\]

Vary with time (earth rotation)

Baseline length

The calibration must determine corrections for R and L independently, which are then transferred to all the products coming out from the correlator.
Gathering Ideas: 2. basic relations of calibration

\[ \langle x_i(t) \cdot x_j^*(t) \rangle = \int C_i(\theta, \phi, t) C_j^*(\theta, \phi, t) s_i(\theta, \phi, t) s_j^*(\theta, \phi, t) d\theta d\phi \]

\( C(t, \theta, \phi) \) is an antenna-based complex quantity. It must be determined for both \( L \) and \( R \) independent systems and this — in practice — means “calibration”

\[ \tilde{C}_i = \overrightarrow{B}_i \overrightarrow{G}_i \overrightarrow{D}_i \overrightarrow{E}_i \overrightarrow{P}_i \overrightarrow{T}_i \overrightarrow{F}_i \]

the order of the terms along the signal path from right to left is

\( F = \text{ionospheric Faraday Rotation}^* \)
\( T = \text{tropospheric effects} \)
\( P = \text{parallactic angle} \)
\( E = \text{antenna voltage pattern} \)
\( D = \text{polarization leakage} \)
\( G = \text{electronic gain} \)
\( B = \text{bandpass response} \)

\text{blue: common to both R & L, purple: independent contributions to C for R & L}
The interferometric product of $C_i C_j = C_{ij}$ can be separated into various homologous pairs

\[ \tilde{C}_i \tilde{C}_j = \tilde{C}_{ij} = \tilde{B}_{ij} \tilde{G}_{ij} \tilde{D}_{ij} \tilde{E}_{ij} \tilde{P}_{ij} \tilde{T}_{ij} \tilde{F}_{ij} \]

N-element interferometer has \( N(N-1)/2 \) \( C_{ij} \) combinations

The values of \( C_{ij} \) may be related to baseline – ID, baseline–length and frequency

In practice, the complex correction can be always divided into

a “gain” - \( G \) - and a “phase” - \( P \) - contribution

The solutions for \( G_i \) and \( P_i \) are “statistical”
\[ V_{i,j}^{\text{true}}(u,v,w,\nu) = V_{i,j}^{\text{obs}}(u,v,w,\nu) \ast G_i \ast G_j \ast e^{-iP_i} \ast e^{-iP_j} \]

Gain (G) and phase (P) are assigned to each antenna in order to have smallest difference between (a – priori known) **ideal** and **real** visibility function.

G variations occur on timescales longer than phase (mostly atmospheric) variations; both are wavelength dependent.

Phase variations make a point source wander around its position.

Ideally the true phase of a given FoV at a given antenna (interferometer) is the superposition of several contributions:

\[ \theta_{\text{true}} = \theta_{\text{source}} + \theta_{\text{geometry}} + \theta_{\text{atmosphere}} + \theta_{\text{instrumentation}} + 2n\pi \]

once solved for “ideal sources” and transferred to “target sources” the only contribution left comes from the target source structure.

on the image plane residual P errors have worse effects than residual G errors.
A successful observation requires proper “scheduling” i.e.

- plan calibration well before the observation
- time spent on calibrators is not wasted (don't forget time for your target source[s])

Calibration sources must sample time variations of both G and P (and B)

- consider coherence time when scheduling calibrators \( \Rightarrow \) time on target source
- strong/weak calibrators require short/long integration time \( \Rightarrow \) dynamic range
- Spectral line obs (spectral work, wide field,...) requires a strong calibration source
How big the FOV needed (further: are there any known strong confusing sources?)

- primary beam
  \[ \theta_{\text{FOV}} \approx \frac{1}{2} \theta_{\text{HPBW}}^{\text{antenna}} \]

- bandwidth smearing
  \[ \theta_{\text{FOV}}^{\text{BW}} \approx \frac{\nu_{\text{obs}}}{\Delta \nu} \theta_{\text{HPBW}}^{\text{synth}} \]

- time smearing
  \[ \theta_{\text{FOV}}^{\text{TA}} \approx \frac{\theta_{\text{HPBW}}^{\text{synth}}}{\omega_{\text{earth}} \tau_a} \]

All these effects are frequency dependent and have different consequences
- large source at high frequency
- weak source at low frequency

Which is the r.m.s. noise level? (play with \( \Delta t \) and \( \Delta \nu \))
An ideal calibration source is:

point-like – amplitude is the same over the whole uv – range

at the pointing centre – phases = zero on all baselines at all times

in case the flux density is constant in time (over many years) then we have a PRIMARY calibrator (3C286, 3C48, 3C147, 3C138, ....)

SECONDARY calibrators may vary

The list of suitable calibration source changes

• at the various observing frequencies
• on the basis of the resolution (e.g. WSRT, VLA, GMRT, MERLIN, EVN, VLBA)

• Calib for instrumental polarization (unpolarized?/parallactic angle sampling)
• Orientation of the E vector (constant E orientation – 3C286, ...)
• Bandpass calibrators: small and strong
Main “canonical steps” in a-priori calibration

Primary calibrator(s) $\Rightarrow$ bootstrapping the amplitudes to the “real scale”

Bandpass calibrator(s) $\Rightarrow$ flattening response with frequency

Secondary calibrator(s) $\Rightarrow$ finding solution suitable for targets

(polarization calibrators) $\Rightarrow$ removing cross-talks

calibration transfer $\Rightarrow$ extending solutions to targets

N.B. [again!] The best a-priori calibration can be achieved through an appropriate scheduling (i.e. think first, then cross your fingers, finally play with data)

then

self – calibration: second order phase (eventually gain as well) variation between calibrator scans may be corrected by using the data on the target source itself
Calibration in practice

Feed a computer with some (obscure) appropriate parameters, and the software does the work for you....

Various software packages with appropriate documentation showing how principles have been implemented. Often pipelines are offered.

It is very important to

- check the results [i.e. solutions] coming out of the various steps
- be able to understand what is good and what has gone wrong! (despite apparent successful operations – as told by software)
- take appropriate actions to fix problems
What is a “visibility”?

at a given observing frequency, for a given pointing (target)

*time – baseline id – length – orientation – amplitude – phase – weight*

0:17:35:10 1-2 3.54 2.68 0.54 0.037 162 8
0:17:35:10 1-3 2.65 8.21 0.77 0.041 –137 6

.....

is the minimum information

(amplitude + phase + weight \* 4 for full Stokes)

(x N if N spectral channels)

a “real” dataset (VLA at 15 GHz)
It is VERY important to inspect your data BEFORE you start with calibration.

There are various ways to inspect raw visibilities:

- **uv-coverage**,  
- **amplitude/phase** vs. time, frequency, baseline-length, ...

let us consider visibilities from a given observation, one polarization hand in spectral line mode.
• amplitude vs. time for a point-like source
Handling real “noise” [data] 4. Inspection of raw data

Amplitudes vs. time

Edit out these!
Amplitude variations across the band mean that the gain is frequency dependent.
Handling real “noise” [data]  4. Inspection of raw data

phases .vs. time

color according to frequency channels
RFI across the band: bad data have to be edited out

- Top panel: phases
- Bottom panel: amplitudes
Example of normalized band pass profile: 128 CHANnels (may be grouped into Ifs) single polarization (RR), single baseline
Normalised band bass profile: a solution for each CHANnel for each antenna

Handling real “noise” [data] 4. Inspection of raw data
Handling real “noise” [data]  4. Inspection of raw data

Amp bandpass solutions have been applied:

Before

After
Once all the data on calibration source have been carefully edited out...
Handling real “noise” [data] 5. Applying a-priori information

raw phases may seem “noise”
Handling real “noise” [data] 5. Applying a-priori information

After P corrections applied, phases show coherence.
Handling real “noise” [data] 5. Applying a-priori information

Solutions must be searched for and found for all the calibration sources.
Antenna-based errors separated into an “amplitude” and a “phase” correction

\[ V_{ij}^{true} = C_i C_j^* \]
\[ V_{ij}^{obs} = G_i G_j V_{ij}^{obs} e^{i(\varepsilon_i - \varepsilon_j)} \]

Closure phase:

\[ \Phi_{ij}^{obs} = \Phi_{ij}^{true} + \varepsilon_i - \varepsilon_j \]
\[ \Phi_{ij}^{obs} + \Phi_{jk}^{obs} + \Phi_{ki}^{obs} = \Phi_{ij}^{true} + \Phi_{jk}^{true} + \Phi_{ki}^{true} \]

Phase solutions \( \varepsilon_i \) must preserve the relation on all possible triangles

Closure amplitude:

\[ \frac{|V_{ij}^{obs}| \cdot |V_{kl}^{obs}|}{|V_{ik}^{obs}| \cdot |V_{jl}^{obs}|} = \frac{|V_{ij}^{true}| \cdot |V_{kl}^{true}|}{|V_{ik}^{true}| \cdot |V_{jl}^{true}|} \]

Gain solutions \( G_i \) must preserve the relation on all possible quartets

Closure phase & closure amplitude are unaffected by antenna gain errors
They are preserved during a-priori & self–calibration
Closure relations provide
(N–2) of phase constraints for N variables (P),
(N–3) of amplitude info for (N-1) variables (normalization)
Many non-independent quantities

However
They do not have Gaussian error distribution
No position or flux density info

Despite all these good news, residual calibration errors may still be present
It is VERY important to inspect your solutions: amplitude corrections, for 2 IF, 2 POL, VLA data derived from a flux density (primary) calibrator and a number of secondary (phase) calibrators.

No smoothing applied to solutions ⇒ Solutions extrapolated to target sources as well.
Scans on calibrators have been highlighted.

Variations at a few % level are OK

Once the a – priori calibration is over, the solutions are extrapolated from calibration sources to targets as well

Smoothing may be useful, at least for amplitudes

Search for phase jumps in phase solutions

Pay attention to target scans close to calibrator scans with problems
Handling real “noise” [data] 6. (double) checking results

phase jump!
simple interp would give wrong phases for target source

180 deg R-L phase jumps in AN13
When A-PRIORI calibration is over.......  

the data can now be coherently **averaged in frequency and in time** [in principle! (and when useful)]

**speeds up all the subsequent data processing** (imaging and self-calibration, further second-order editing, plotting, etc.)

**may be dangerous**, phases are likely to need further adjustment before averaging (true for weak targets)

**absolute position referred to secondary calibrators**
Fourier Inversion & Imaging:

The observation samples the Visibility Function:

\[
V_{i,j}^{\text{obs}}(u,v) = V_{i,j}^{\text{true}}(u,v) \cdot \sum_k (\delta(u-u_k)\delta(v-v_k))
\]

the use of the FFT requires “regular” gridding, i.e. convolution and resampling

FT theory relates the sampling of the image plane and that of the uv – plane
Ideally, if $U_{\text{max}}$ is the largest baseline, the image plane needs a sampling

$$\Delta x < \frac{1}{2 U_{\text{max}}} \quad [\text{rad}]$$

How big the image needs to be?

Consider the FoV:

- Primary Beam
- Bandwidth Smearing
- Time smearing

The sky also looks very different at various frequencies. In general, if there are strong confusing sources, their sidelobes at the pointing centre will reveal the position of such “disturbances” that can be removed.
- primary beam  \[ \theta_{FOV} \approx \frac{1}{2} \theta_{HPBW}^{antenna} \]

- bandwidth smearing  \[ \theta_{FOV}^{BW} \approx \frac{\nu}{\Delta \nu} \theta_{HPBW}^{synth} \]

- time smearing  \[ \theta_{FOV}^{TA} \approx \frac{\theta_{HPBW}^{synth}}{\omega_{earth} \tau_a} \]
How a dirty image may look like
**Fourier inversion (dirty image, beam)**
A field of view appropriate to the target source is imaged and then cleaned.

Image parameters (pixel – size, image – size, weighting, # CC, ...) are chosen on the basis of the uv-coverage (min – max baseline, time on source, # of visibilities, ...) and of the aims of the observation

Cleaning removes the PSF pattern and its side-lobes leaving a residual image on which the CC are restored after convolution with a clean beam

**How cleaning works** ...

At the beginning, residual errors limit the sensitivity and clean cannot be deep. The CC file as model for self – calibration

**Self – Calibration**
The visibilities are modified according to closure relations.

Despite all these good news, residual calibration errors may still be present
Antenna-based errors separated into an “amplitude” and a “phase” correction

\[ V_{ij}^{\text{true}} = C_i C_j^* \quad V_{ij}^{\text{obs}} = G_i G_j V_{ij}^{\text{obs}} e^{i(\varepsilon_i - \varepsilon_j)} \]

Closure phase:

\[ \Phi_{ij}^{\text{obs}} = \Phi_{ij}^{\text{true}} + \varepsilon_i - \varepsilon_j \]

\[ \Phi_{ij}^{\text{obs}} + \Phi_{jk}^{\text{obs}} + \Phi_{ki}^{\text{obs}} = \Phi_{ij}^{\text{true}} + \Phi_{jk}^{\text{true}} + \Phi_{ki}^{\text{true}} \]

Phase solutions \( \varepsilon_i \) must preserve the relation on all possible triangles

Closure amplitude:

\[ \frac{|V_{ij}^{\text{obs}}| \cdot |V_{kl}^{\text{obs}}|}{|V_{ik}^{\text{obs}}| \cdot |V_{jl}^{\text{obs}}|} = \frac{|V_{ij}^{\text{true}}| \cdot |V_{kl}^{\text{true}}|}{|V_{ik}^{\text{true}}| \cdot |V_{jl}^{\text{true}}|} \]

Gain solutions \( G_i \) must preserve the relation on all possible quartets

Closure phase & closure amplitude are unaffected by antenna gain errors

They are preserved during a-priori & self – calibration
Phase (gain) corrections are calculated on the basis of a least square difference between input visibilities and [CC] model.

A few parameters need to be adjusted: UVRANGE, SOLINT, SNR, ...

How much flux density is in the model? Does it represent the full flux density in the field?

“absolute position is lost” [not completely true, accuracy of the order of the pixel size]
- Inspecting solutions
- Reference antenna has zero phase correction
  - No absolute position info.
- Corrections up to 150° in 14 minutes
- Typical coherence time is a few minutes
- Consider messages on “good” and “failed” soln's

Then new image with the corrected dataset, and new CC model
Inspecting solutions

Corrections are reduced to $40^\circ$ in 14 minutes

Observation now quasi-coherent

Next: shorten solution interval to follow troposphere even better

Then new image with the corrected dataset, and new CC model with more details (flux density)
Improving images: 11. double checking self-calibration

- Inspecting solutions
- 10-second solution intervals
- Corrections look noise-dominated
- Expect little improvement in resulting image

- Then new image will be the “final” one
- unless we want to try amplitude self–cal
Improving images: 11. double checking self-calibration

Amplitude self-calibration:
yellow is after, pink is before

- Normalize amplitude solutions
- Choose appropriate UVRANGE
The **final image** may contain as many CC as the total # of visibilities [...] 

the r.m.s. noise in the image plane should then reach the “thermal noise” reported on telescope (interferometer) user manual

at low frequencies, the FOV is large and then there are many sources in a large number of pixels (e.g. 25 Mpix): slow of facets, may be more difficult to reach the thermal noise

Residual calibration (non closing) errors appear as distortions in the image

Hunting for hidden bad data
point source 2005+403

- process normally
- self-cal, etc.
- introduce errors
- clean

13 scans over 12 hours

6-fold symmetric pattern due to VLA “Y”

10% amp error all ant 1 time
rms 2.0 mJy

no errors:
max 3.24 Jy
rms 0.11 mJy

Improving images: 11. double checking self-calibration
Improving images: 11. double checking self-calibration

10 deg phase error 1 ant
1 time
rms 0.49 mJy

20% amp error 1 ant
1 time
rms 0.56 mJy

anti-symmetric ridges

symmetric ridges
10 deg phase error 1 ant
all times
rms 2.0 mJy

20% amp error 1 ant
all times
rms 2.3 mJy

rings – odd symmetry
rings – even symmetry

Improving images: 11. double checking self-calibration
Once you have a “perfectly calibrated” dataset...

**produce the final image** (play with resolution, weighting, etc.)

**perform image analysis** (noise, flux density, size,...)
GAME OVER

it's time for astrophysics