Mem. S.A.It. Vol. 75, 282

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Investigation of a new approach to reflector profile retrieval

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Abstract. The retrieval of the surface profile of a reflector antenna is an important issue, especially for radioastronomical applications. Microwave holography retrieves the reflector profile starting from a set of measured far-field data. The main step of this technique is the computation of the induced currents on the reflector surface. This requires the solution of a linear inversion problem which is strongly ill-conditioned. The aim of this work is to investigate a new solution technique for this linear inversion problem based on the SVD. This new technique supplies a flexible regularization scheme, able to take into account also the noise level of the data. Some numerical results are obtained, starting from a simulated far-field.

1. Introduction

The gain (and resolution) of a reflector antenna depends heavily on the accuracy of the reflector surface. Therefore the retrieval of the surface profile is a key step in the reflector antenna test. Moreover, for many applications, including all radio-astronomical ones, it is usually required that the profile is checked periodically, in order to preserve the antenna resolution. This is even most important for reflector antennas with active surfaces, such as the Noto radiotelescope (see Fig. 1) and the SRT, which are the present state-of-the-art.

Up to now, many different techniques have been proposed and implemented for the surface diagnostics. Among them, those based on microwave holography allow one to create an accurate map of the reflector surface profile. Microwave holography, starting from a measurement of a part of the antenna pattern (main lobe and first side lobes), calculates the induced currents on the reflector surface. The comparison between the calculated profile and the nominal profile allows one to find the deformations on the reflector.

Here we want to focus on the computation of the induced current, which is a critical point since it is an inverse problem, and therefore strongly ill-conditioned. Moreover, only a very limited amount of information is available on the antenna pattern. As a consequence, the solution must be sought using some regularization procedures. Up to now, this problem has been solved with an extrapolation procedure, based on a simple Fourier transform inversion (Rahmat-Samii 1985). This is quite efficient computationally, but is limited to the paraxial case, and performs an implicit regularization only if the measurement and retrieval points are chosen at the Nyquist rate. The aim of this

Fig. 1. Noto radiotelescope, a 32-m reflector antenna with active surface.

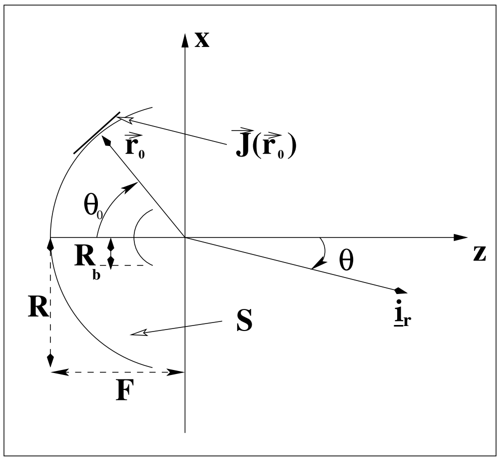

Fig. 2. Reflector geometry.

work is to introduce and investigate a new solution technique for this linear inversion problem, based on the singular value decomposition (SVD). The main advantages are:

- no limitation to the paraxial case;
- no constraint on the measurement and reconstruction points;
- a flexible regularization scheme, able to take into account also the data noise level;
- a high computational efficiency, at least for repetitive measurements.

Since we are interested in the reflector profile, or, in other words in a scalar quantity, only one far-field component will be sufficient to retrieve an induced current component. We choose the field co-polar component because this assures the best signal-to-noise ratio for the data.

2. Construction of the model

The relationship between the induced current on the reflector $\underline{J}\left(\underline{r}_0\right)$ and the far-field pattern $\underline{F}(\theta,\phi)$ is well-known (Rahmat-Samii 1985). We will consider only the field co-polar component, given by

$$F_c(\theta,\phi) = \underline{i}_c \cdot \int_S \underline{J}\left(\underline{r_0}\right)\exp\left(j\beta\underline{i}_r \cdot \underline{r}_0\right)dS_0 \quad (1)$$

where $\underline{i}_r$ points toward the direction (θ,ϕ) and $\underline{r}_0$ is the variable point on the reflector (see Fig. 2). The vector

$$\underline{i}_c = \underline{i}_\theta cos\phi - \underline{i}_\phi sin\phi \quad (2)$$

allows one to determine the field co-polar component. In this case eq. (1) must be seen as an equation where $\underline{J}\left(\underline{r}_0\right)$ is the unknown, and more exactly as a linear Fredholm integral equation of the first kind (Hansen 1998; Bertero et al. 1985). Its inversion is a severely ill-posed problem because of a very smooth kernel and a small domain S (Hansen 1998). It follows that eq. (1) can not be simply inverted, but it is necessary to apply a regularization procedure, which generally needs a different solution definition, also in view of the fact that the measured $F_c(\theta,\phi)$ will include an unavoidable noise. Moreover it must be considered that the data ($F_c(\theta,\phi)$ values) are measured only in a limited number of points along the antenna main lobe and, if necessary, along the first side lobes. Then, the problem has to be formulated as a discrete data inverse problem (Bertero et al. 1985). Let $\left(\theta_p,\phi_p\right)$ be the set of measurement points and $L_p[\;]$ the functional which supplies $F_c\left(\theta_p, \phi_p\right)$ as in eq. (1), with $p = 1, \ldots, N$. If the f_p are the measured values then:

$$f_p = L_p[\underline{J}_T] + n_p = {f_p}^T + n_p \tag{3}$$

where $\underline{J}_T$ is the true current, ${f_p}^T$ the field's "true" value and n_p is the noise on the p-th measure. Defining the numerical vectors $\underline{f}$, $\underline{f}^T$, $\underline{n}$ and a norm of the vector $\underline{x}$: $\|\underline{x}\|^2 = \sum_{p=1}^{N} |x_p|^2$, we can find a solution $\underline{J}_{LS}$ such that:

$$\|\underline{f}^{LS} - \underline{f}\|^2 minimum \tag{4}$$

where ${f_p}^{LS} = L_p\left[\underline{J}_{LS}\right]$, which is a "least squares" solution (to get a deeper understanding from a mathematical point of view, see Bertero et al. 1985, 1987). The main point is that this discrete data problem is well-posed but it turns out to be heavily ill-conditioned (Bertero et al. 1985) because eq. (1) is ill-posed. Therefore the solution of the problem must not only minimize eq. (4), but also have a bounded condition number. Up to now the technique used in Rahmat-Samii (1985) replaces eq. (1) with a Fourier Transform by a paraxial approximation and retrieves the currents with an extrapolation algorithm (Papoulis 1975). If the data (and reconstruction) points are taken at the Nyquist rate (as required by the FFT) the problem is regularized. In this way the actual resolution depends on the extension of the measurement domain, but not on the noise level. Moreover, the complete profile must be obtained by an interpolation completely independent of the currents' reconstruction procedure.

In this work the solution approach is instead to formulate a linear system equivalent to eq. (3), where the unknown current is discretized according to the required "graphical" resolution and the "least squares" solution is regularized using the SVD. As we will see, the SVD analysis not only allows one to separate the actual resolution of the solution (i.e., the amount of independent information obtainable) from the graphics resolution of the solution, but also allows one to tie the actual resolution to the data noise (Hansen 1998). The SVD calculation of a great linear system as eq. (3) is very heavy but this problem can be overcome using some peculiar features of radio-astronomical reflectors. First, they are axis-symmetric, and therefore both the currents and the pattern are better represented as Fourier Series. Moreover they are large and focusing, so the range of θ off the reflector boresight is very small. Actually the vectorial nature of the current leads to a weak coupling between different harmonics which rises when the focal diameter ratio (f/D) is small. However, such coupling causes negligible differences on the profile retrieval, about $10^{-4}\lambda$, and so it can be neglected. Letting $C_p\left(\theta_0\right)$ be the decoupled Fourier harmonics of the current and $t_p\left(\theta\right)$ the Fourier harmonics of the field (suitably normalized), eq. (1) becomes

$$t_p\left(\theta\right) \cong (j)^p \cdot \int_{R_b}^{R} 4\cos\left(\frac{\theta_0}{2}\right) J_p\left(\beta\rho\sin\theta\right) \exp\left(j\beta z\left(\rho\right) cos\theta\right) C_p\left(\theta_0\right) \rho \, d\rho \tag{5}$$

where R_b and R are the blocking and external radii of the reflector, (see Fig.1), J_p is the Bessel function of the first kind, $z\left(\rho\right)$ is the reflector nominal profile, and $\theta_0 = \theta_0\left(\rho\right)$ is the angle measured from the feed boresight. Eq. (5), using a suitable discretization, supplies the linear equations' system (for each p-th harmonics) which allows one to calculate the unknown current.

3. Regularized solution

The Fredholm eq. (5) can be transformed into a linear system by representing $C_p\left(\theta_0\right)$ as a piecewise-constant function and sampling both sides at the measurement points. This is not a direct task, since the computation of $t_p\left(\theta\right)$ requires the measurement points to be located on concentric circles (antenna framework), while typically measured data are available on azimuth-elevation coordinates (terrestrial framework). It is however possible to transform between the two frameworks, using the limited band of the reflector field (Bucci & Franceschetti 1987).

The resulting linear system $\underline{A} \cdot \underline{c} = \underline{t}$ can be solved using a regularization procedure based on the SVD (Hansen 1998) of the $\underline{A}$ matrix

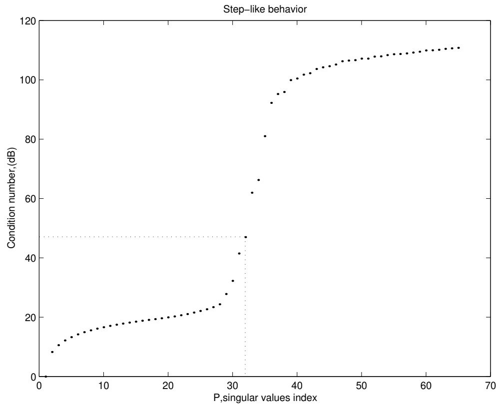

Fig. 3. Step-like behavior of the condition number.

$$\underline{A} = \underline{U}^H \cdot diag\,(\sigma_1, \ldots, \sigma_N) \cdot \underline{V}. \quad (6)$$

After computing the SVD of the system matrix, the so-called "principal components solution" can be evaluated as

$$\underline{C_{pc}} = \underline{R_p} \cdot \underline{t} \quad (7)$$

where

$$\underline{R_p} = \left[\underline{V}^H \cdot diag\left(\frac{1}{\sigma_1}, \ldots, \frac{1}{\sigma_P}, 0, \ldots, 0 \right) \cdot \underline{U} \right]$$

and P, the effective rank of $\underline{A}$, is suitably chosen in order to find the better trade-off between solution stability and accuracy. More precisely, since the condition number of the $\underline{A}$ has a step-like behavior when the index P of the σ_P sequence increases (see Fig. 3), the best choice is a P-value just before the step (Hansen 1998). In this way we get the best accuracy, while the condition number of the solution remains bounded.

Some tests have been performed at 11.4 GHz on a dual-reflector having the Noto antenna geometry: a diameter of 32 m, a sub-reflector diameter of 3.2 m and a focal length diameter ratio of 0.32. The reflector far-field has been simulated with GRASP* CAD (Ticra) on a grid of 64 × 64 measurement points in the terrestrial framework (observation window between $-1\overset{\circ}{.}25$ and $1\overset{\circ}{.}25$) and then interpolated in the antenna framework (observation window with $\theta_{MAX} = 1\overset{\circ}{.}65$) using the functions (Bucci & Di Massa 1988). The surface profile of the reflector has been retrieved using eq. (5) and compared with a reference one. Two different "point shape" deformations, located on the reflector at $x = -9.6$ m, $y = -9.6$ m, have been considered: 6 mm and 3 mm. The results are shown in Figs. 4a and b, respectively. In both cases, using the current harmonics up to the 36-th order, the deformations have been localized and retrieved, though their amplitude is equal to 66% of the actual deformations (see Figs. 5a,b).

4. Conclusions

The technique based on the SVD-approach supplies a new electromagnetic model for the diagnostic of large reflector antennas starting from a small set of far-field data. From a practical point of view the greatest amount of retrieval is to compute eq. (7), where, however, it's important to emphasize that the $\underline{R_p}$ matrix can be computed only once and stored (since the measurement set-up does not change), thus reducing the complexity of the approach. It is worth noting that we could equally well start from a set of Fresnel-zone data, since the intermediate field is still axi-symmetric, and allows to write an equation of the same kind as eq. (1). Since the Fresnel-field is stronger by orders of magnitude than the far-field, this significantly improves the SNR of the data and therefore the reconstruction accuracy. This extension is in progress.

Acknowledgements. The authors would like to thank the Electromagnetics Laboratory (Università di Firenze) for providing the Grasp simulations.

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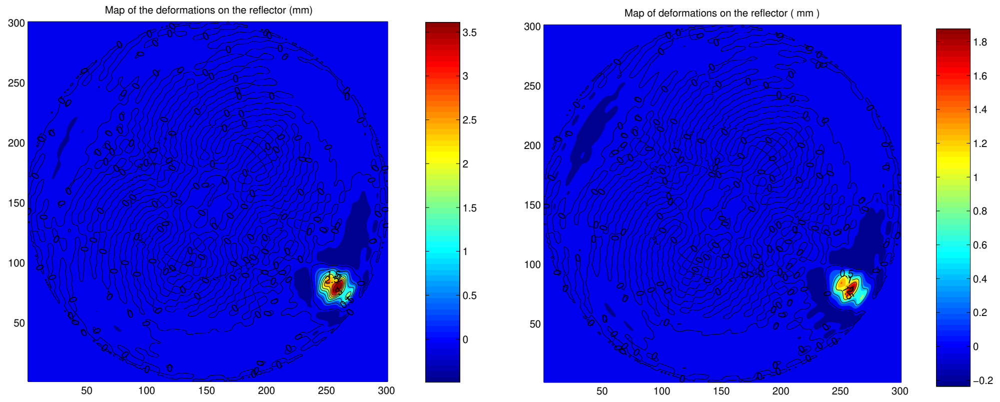

Fig. 4. a) Reflector with a 6 mm "point shape" deformation; b) Reflector with a 3 mm "point shape" deformation (x along vertical axis, y along horizontal axis).

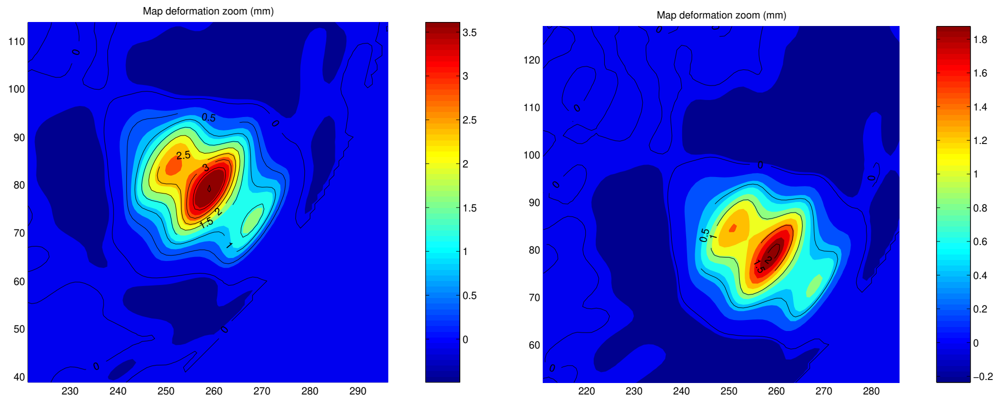

Fig. 5. a) Reconstructed deformation zoom, 4 mm of the actual deformation (6 mm); b) Reconstructed deformation zoom, 2 mm of the actual deformation (3 mm).

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