Thomson scattering:

"It is the scattering of electromagnetic radiation by a free non-relativistic charged particle.

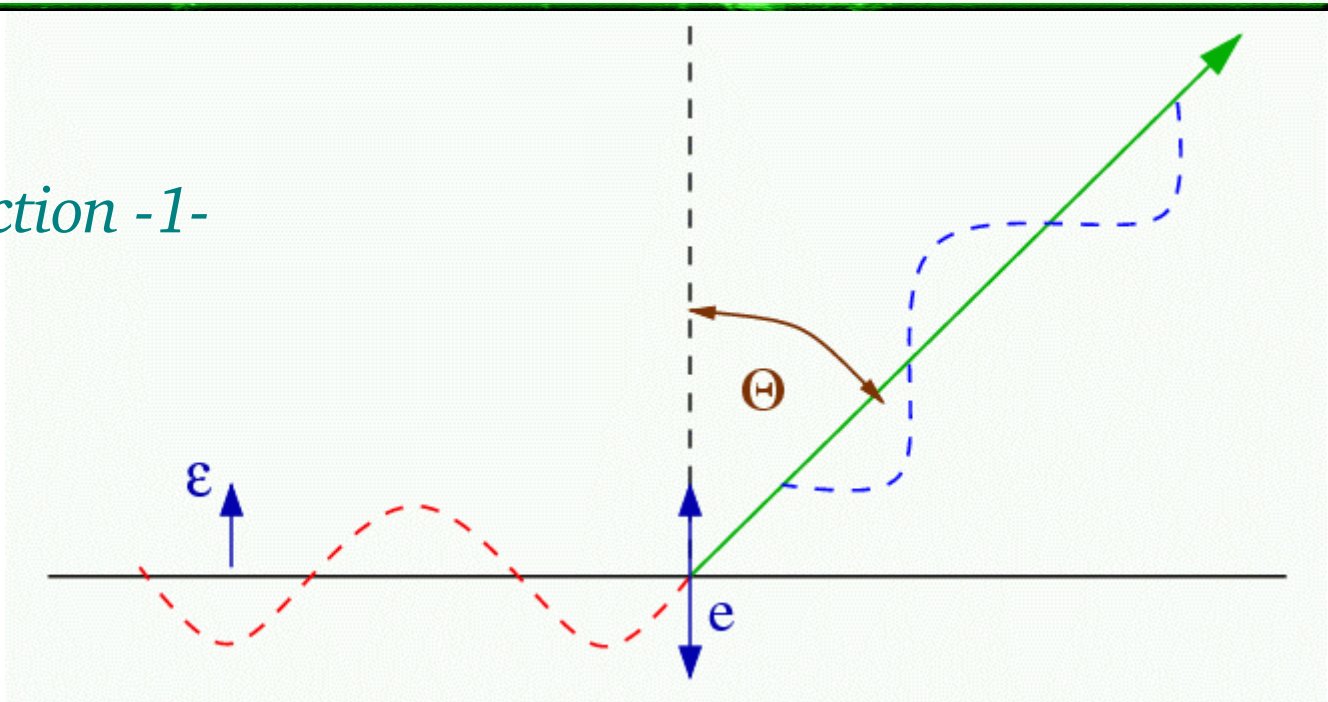
The electric and magnetic components of the incident wave accelerate the particle. As it accelerates, it, in turn, emits radiation and thus, the wave is scattered. Thomson scattering is an important phenomenon in plasma physics and was first explained by the physicist

The main cause of the acceleration of the particle will be due to the electric field component of the incident wave. The particle will move in the direction of the oscillating electric field, resulting in electromagnetic dipole radiation. The moving particle radiates most strongly in a direction perpendicular to its motion and that radiation will be polarized along the direction of its motion. Therefore, depending on where an observer is located, the light scattered from a small volume element may appear to be more or less polarized."

Thomson scattering

photon-electron interaction -1-

$h\nu \ll m_e c^2$



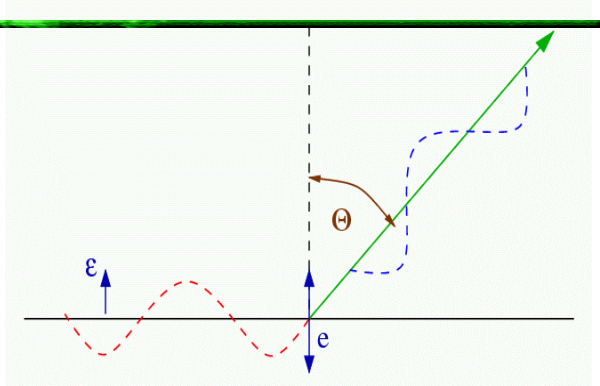
– *for a non relativistic particle, the incoming (low-energy) photons can be expressed as a continuous e-m wave and the magnetic field ($H_o = E_o$) can be "neglected" (i.e. studied as a consequence of the E field behaviour).*

– *the average incoming Poynting flux is* $<|S|> = cE_o^2/8\pi$

– *the charge feels a force due to a linearly polarized incoming wave:*

– *the energy of the scattered radiation is the same as the incoming wave*

Thomson scattering (2)



– *the force is due to a linearly polarizesd wave:*

$$\vec{F} = e\hat{\varepsilon}E_o \sin\omega_o t$$

$$\vec{d} = e\vec{r} \quad \rightarrow \quad \ddot{\vec{d}} = e\ddot{\vec{r}} = \frac{e^2 E_o}{m}\hat{\varepsilon}\sin\omega_o t$$

– *the dipole moment is*

$$\vec{d} = -\left(\frac{e^2 E_o}{m\omega_o^2}\right)\hat{\varepsilon}\sin\omega_o t = \vec{a}(t)$$

– *which describes a dipole with amplitude* $\vec{d}_o = -\left(\frac{e^2 E_o}{m\omega_o^2}\right)\hat{\varepsilon}$

– *taking into account the Larmor's formula, the time averaged emitted W is*

$$\left\langle \frac{dW}{d\Omega} \right\rangle = \left\langle \frac{2}{3}\frac{\ddot{\vec{d}}^2}{c^3} \right\rangle = \left\langle \frac{e^4 E_o^2}{8\pi m^2 c^3}\sin^2\Theta \right\rangle = \langle S \rangle \frac{d\sigma}{d\Omega}$$

– *where* $\frac{d\sigma}{d\Omega} = \frac{e^4}{m^2 c^4}\sin^2\Theta = r_o^2 \sin^2\Theta$

is the differential cross section for the scattering

Particle acceleration

(due to incoming radiation)

$$\boldsymbol{a}(t) = \frac{e\varepsilon\, E_o}{m_e} \sin\omega\, t$$

the emitted power by the scattering particle is:

$$-\frac{d\varepsilon}{dt} = \frac{2e^2}{3c^3}\, a^2 = \frac{2e^2}{3c^3}\,\frac{e^2 E_o^2}{m_e^2}\,\frac{1}{2} = \frac{e^4 E_o^2}{3m_e^2 c^3}$$

from this equation we define the electron Thomson cross section by integrating over all the angles:

$$\sigma_T = \frac{< d\varepsilon/dt >}{< |\boldsymbol{S}| >} = \frac{8\pi}{3}\left(\frac{e^2}{m_e c^2}\right)^2 = \frac{8\pi}{3}\, r_0{}^2 = 6.65 \times 10^{-25} \quad \mathrm{cm}^{-2}$$

classical electron radius is

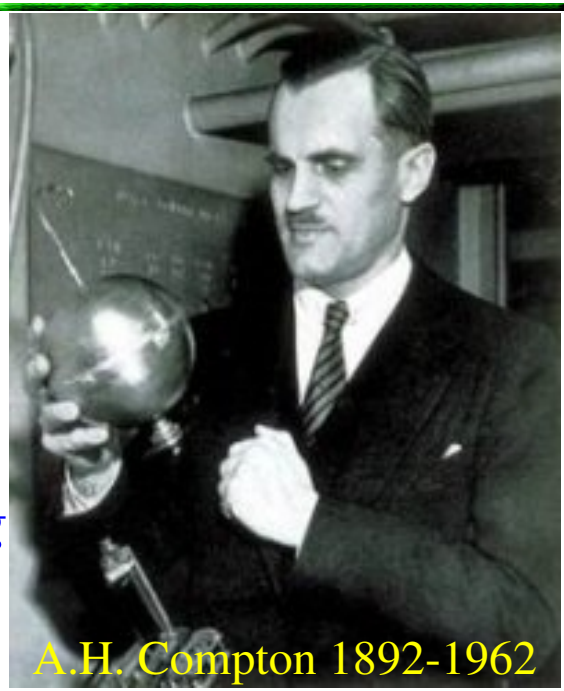
$$r_0 = \frac{e^2}{m_e c^2} = 2.82 \times 10^{-13}\,\mathrm{cm}$$

Back to the "geometric problem" the angular distribution of the radiation is given

$$< \frac{d\varepsilon}{d\Omega} > = \frac{\ddot{\boldsymbol{d}}^2}{4\,\pi\, c^3}\, \sin^2\Theta$$

and this means that each individual scatter produces polarized radiation ***Cross section independent from frequency*** *(breaks down at high frequencies when the classical physics need quantum mechanics)*

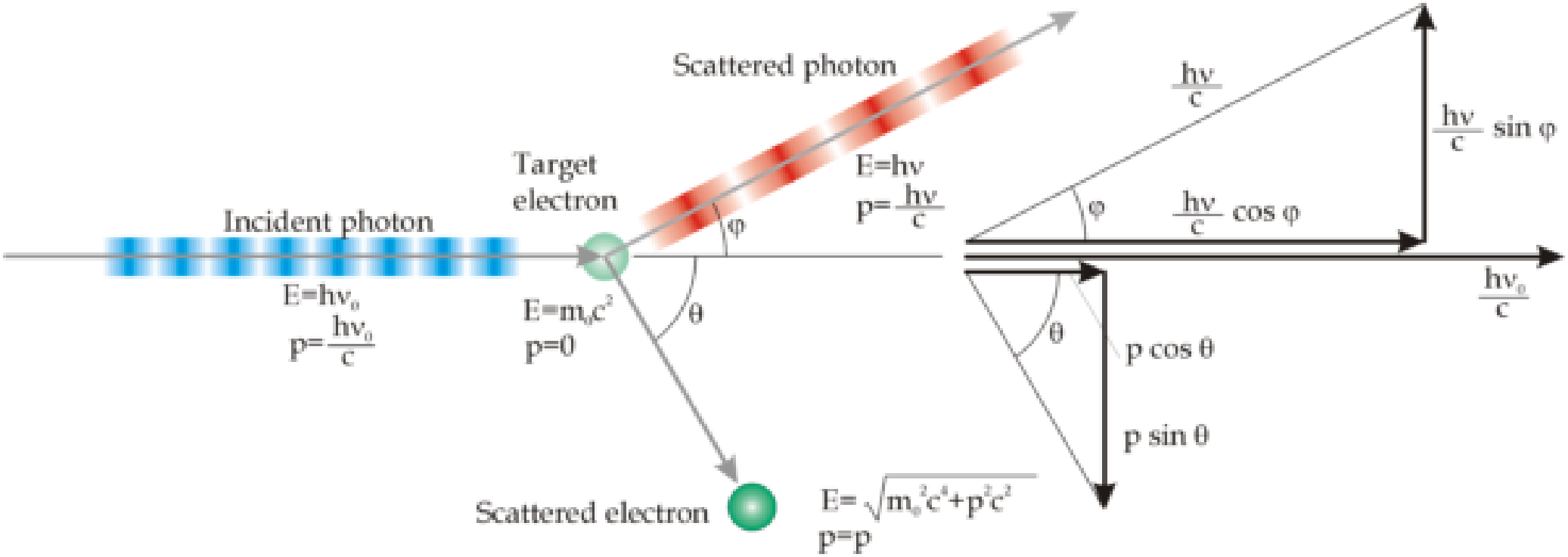
Compton scattering

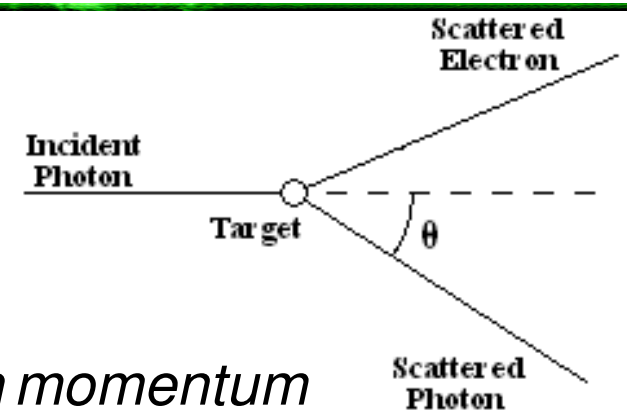

A.H. Compton 1892-1962

"In physics, Compton scattering or the Compton effect, is the decrease in energy (increase in wavelength) of an X-ray or gamma ray photon, when it interacts with matter.
Inverse Compton scattering also exists, where the photon gains energy (decreasing in wavelength) upon interaction with matter. The amount the wavelength increases by is called the Compton shift. Althoughnuclear compton scattering exists, what is meant by Compton scattering usually is the interaction involving only the electrons of an atom.
Compton effect was observed by Arthur Holly Compton in 1923, for which he earned the 1927 Nobel Prize in Physics.

The effect is important because it demonstrates that light cannot be explained purely as a wave phenomenon.
Thomson scattering, the classical theory of charged particles scattered by an electromagnetic wave, cannot explain any shift in wavelength. Light must behave as if it consists of particles in order to explain the Compton scattering."

Compton scattering





energy conservation:

$$h\nu_i + m_e c^2 = h\nu_f + \sqrt{p_e^2 c^2 + m_e^2 c^4}$$

momentum conservation:

$$\vec{p}_i = \vec{p}_f + \vec{p}_e \quad \rightarrow \quad p = E/c = h\nu/c \quad \rightarrow \quad \textit{photon momentum}$$

$$\vec{p}_e = \vec{p}_i - \vec{p}_f \quad \rightarrow \quad p_e^2 = (\vec{p}_i - \vec{p}_f)\cdot(\vec{p}_i - \vec{p}_f) = p_i^2 + p_f^2 - 2\,p_i\,p_f \cos\theta$$

$$p_e^2 = h^2\nu_i^2 + h^2\nu_f^2 - 2h^2\nu_i\nu_f\cos\theta$$

multiply by c^2 then take the square of energy conservation

$$p_e^2 c^2 + m_e^2 c^4 = h^2\nu_i^2 + h^2\nu_f^2 + m_e^2 c^4 + 2h\nu_i m_e c^2 - 2h\nu_f m_e c^2 - 2h^2\nu_i\nu_f$$

compare the two by highlighting $p_e^2 c^2$ *we get*

$$-2h^2\nu_i\nu_f\cos\theta = 2h\nu_i m_e c^2 - 2h\nu_f m_e c^2 - 2h^2\nu_i\nu_f$$

$$2h^2\nu_i\nu_f(1-\cos\theta) = 2h m_e c^2(\nu_i - \nu_f) \quad \rightarrow \quad \frac{1-\cos\theta}{m_e c^2} = \frac{\nu_i - \nu_f}{h\nu_i\nu_f} = \frac{1}{h}\left(\frac{1}{\nu_f} - \frac{1}{\nu_i}\right)$$

$$\frac{1}{\nu_f} - \frac{1}{\nu_i} = \frac{\lambda_f}{c} - \frac{\lambda_i}{c} = \frac{h}{m_e c^2}(1-\cos\theta)$$

$$\lambda_f - \lambda_i = \frac{h}{m_e c}(1-\cos\theta) = \lambda_o(1-\cos\theta)$$

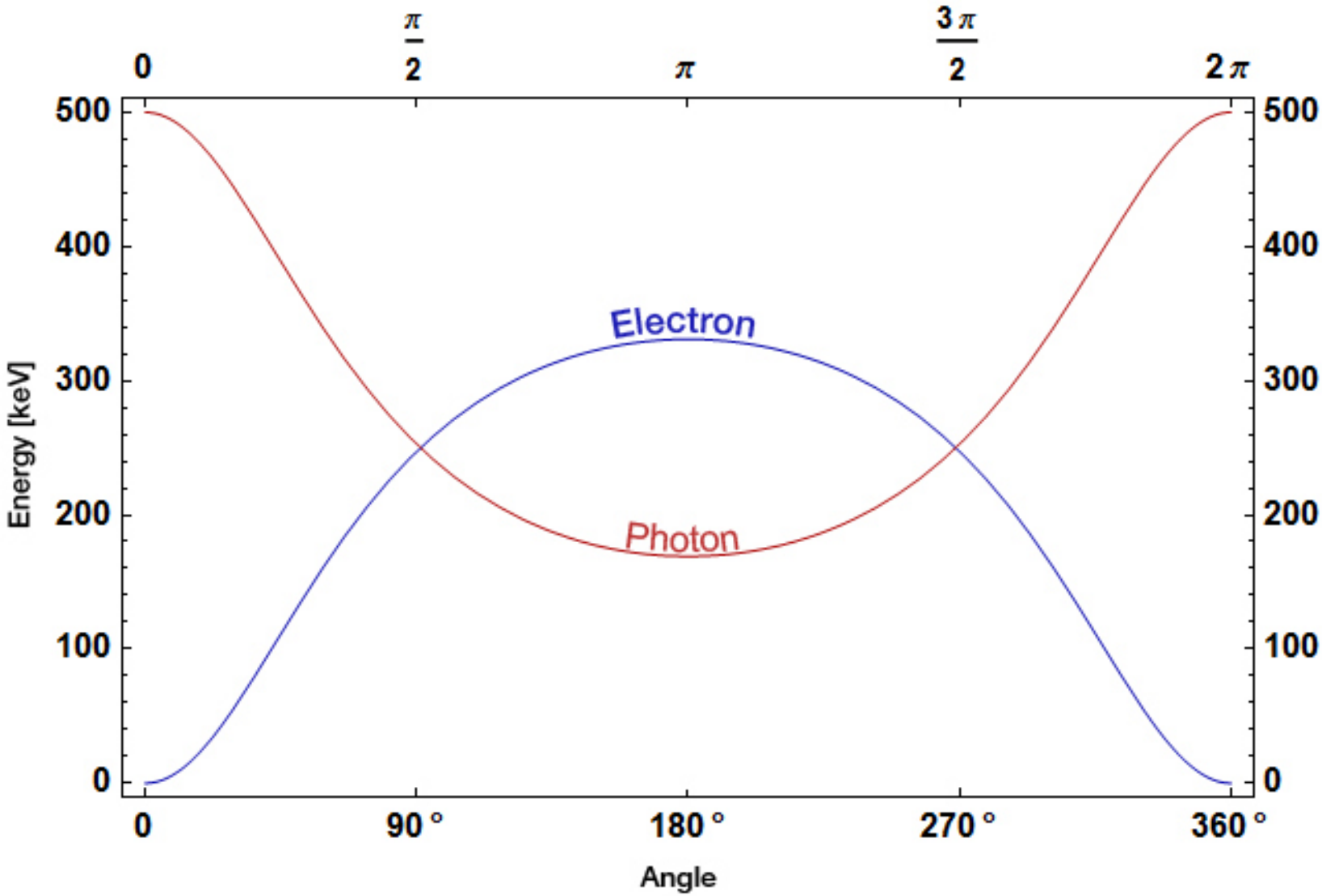
Compton wavelength for electrons

$$\lambda_o = \frac{h}{m_e c} = 0.02426\,A$$

Compton scattering

a Java applet showing the geometry of the scattering can be found at

http://www.student.nada.kth.se/~f93-jhu/phys_sim/compton/Compton.htm



Compton scattering

a Java applet showing the geometry of the scattering can be found at

http://www.student.nada.kth.se/~f93-jhu/phys_sim/compton/Compton.htm

$$h\nu_e = \frac{h\nu_i}{1 + h\nu_i(1-\cos\theta)/m_e c^2}$$

$$\lambda_e - \lambda_i = \frac{h}{m_e c}\cdot(1-\cos\theta) = \lambda_o(1-\cos\theta) = 0.02426\cdot(1-\cos\theta) \quad [\mathrm{A}]$$

Final energy of the scattered photon is a function of:

– *initial energy of the photon*
– *scattering angle*

The photon always looses energy, unless $\theta = 0$, *and the scattering is closely elastic when* $\lambda \gg \lambda_c$ *(i.e.* $h\nu \ll m_e c^2$ *)*

When the photons involved in the collision have large energies, the scattering become less efficient and quantum electrodynamics effects reduce the cross section:
the Thomson cross section becomes the Klein-Nishina cross section

In case $h\nu \sim m_e c^2$, *the probability of interaction decreases, also as a function of the scattering angle; additional constraints come from relativistic quantum mechanics;*

if we use $x \equiv \dfrac{h\nu}{m_e c^2}$

$$\sigma = \sigma_T \frac{3}{4}\left[\frac{1+x}{x^3}\left(\frac{2x(1+x)}{1+2x} - \ln(1+2x)\right) + \frac{1}{2x}\ln(1+2x) - \frac{1+3x}{(1+2x)^2}\right]$$

known as the ***Klein – Nishina*** *cross section;*

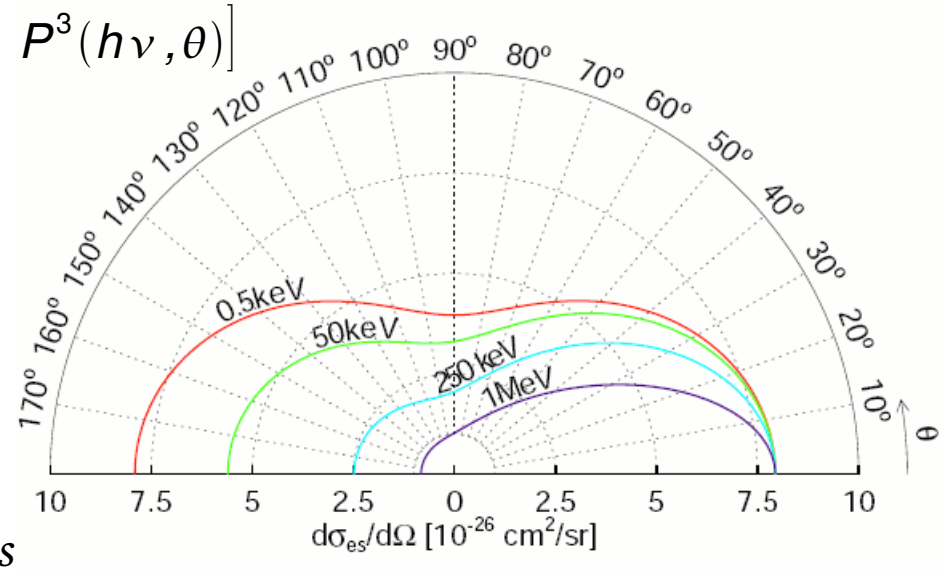
$$\text{if } x \gg 1 \quad \rightarrow \quad \sigma = \sigma_T \frac{3}{8}\frac{1}{x}\left(\ln 2x + \frac{1}{2}\right)$$

it originates from a differential cross section based on the scattering angle

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} r_e^2 \left[P(h\nu,\theta) - P^2(h\nu,\theta)\sin^2\theta + P^3(h\nu,\theta)\right]$$

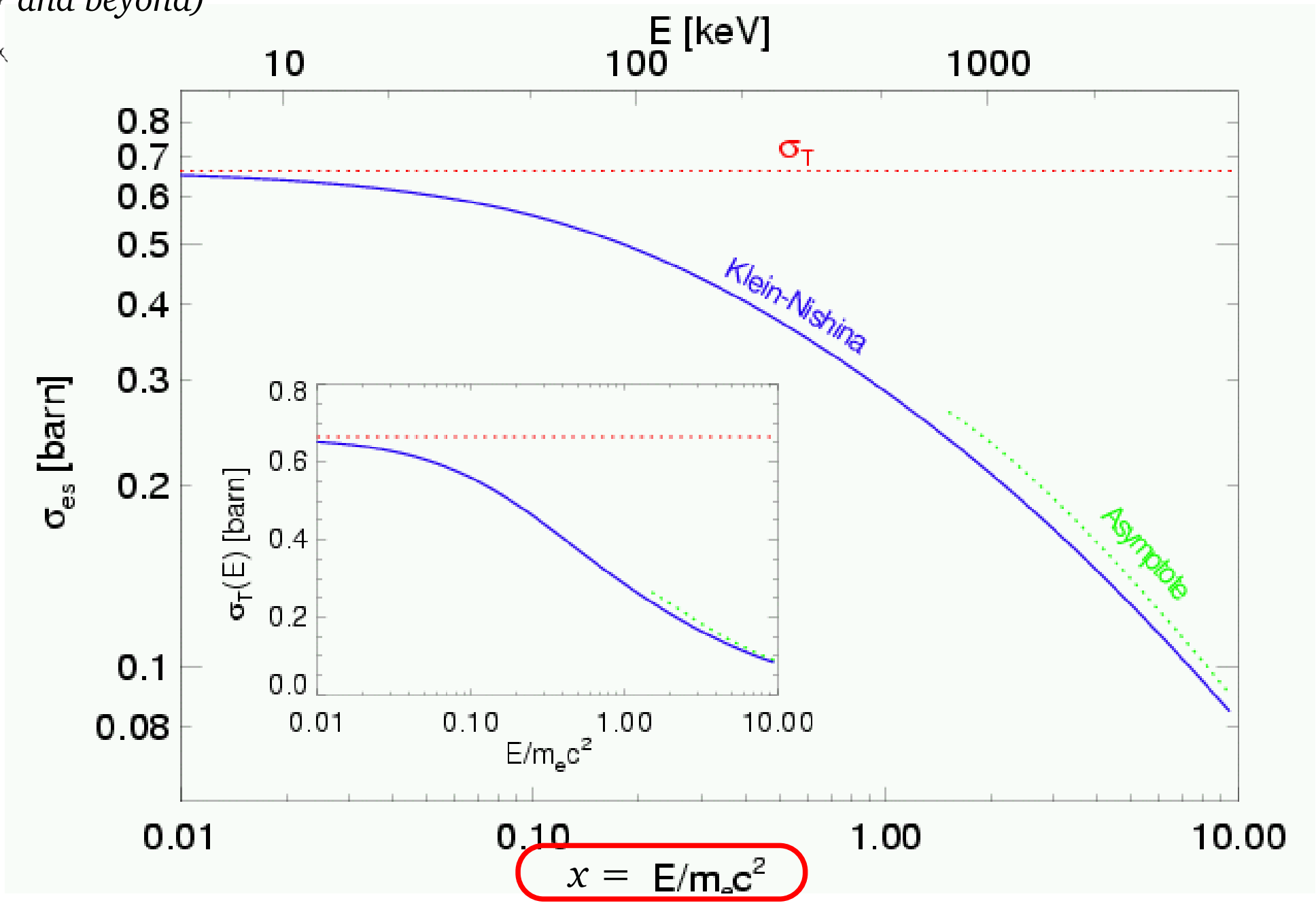
where the probability is defined as

$$P(h\nu,\theta) = \frac{1}{1 + \dfrac{h\nu}{m_e c^2}(1-\cos\theta)}$$



and the Klein – Nishina cross section becomes the Thomson cross section at low photon energies

*The **Klein – Nishina** cross section is relevant at high photon energies (Hard X-Rays and beyond)*



Inverse Compton scattering

Moving, energetic (relativistic), electrons "cool down" by increasing the energy of seed photons.

To make computations simpler, it is possible to consider the centre of momentum frame where the photon energy is much less than $m_e c^2$ and then σ_T can be used.

In the electron rest frame we have Thomson scattering

1. Lab system ⇨ electron rest frame:

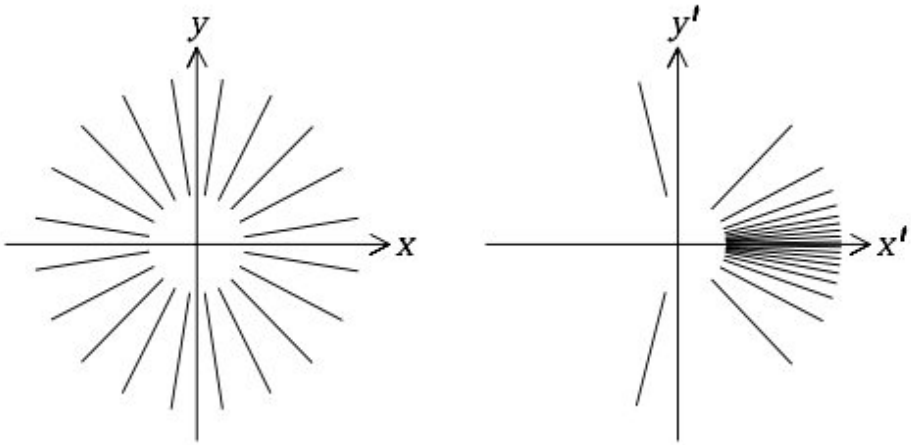
$$h\nu_e = h\nu_L \, \gamma \, (1 - \beta \cos\theta)$$

2. Thomson scattering in electron rest frame

$$h\nu_e' = h\nu_e$$

3. Electron rest frame ⇨ Lab system

$$h\nu_L' = h\nu_e' \, \gamma \, (1 + \beta \cos\theta_1')$$



i.e. ⇨

$$h\nu_L' \approx \gamma^2 h\nu_L$$

Very efficient energy transfer from electron to photon!

Inverse Compton scattering

The scattering angle is important for the energy of the outgoing electron:

1 - maximum energy gained by the electron if $\theta = \pi$ *and* $\theta_1' = 0$

- *in the electron rest frame the photon is blue – shifted (face on collision)*

2 – minimum energy gained by the electron if $\theta = 0$ *and* $\theta_1' = \pi$

- *in the electron rest frame the photon is red – shifted (end on collision)*

the maximum energy that can be gained is (averaged over all angles):

$$\varepsilon' \approx \frac{4}{3}\gamma^2 \varepsilon$$

or, in terms of frequency

$$\boxed{\nu' \approx \frac{4}{3}\gamma^2 \nu}$$

In case it is possible to measure both the initial and final photon energy/frequency, then γ^2 *can be deduced*

$$\boxed{\gamma \approx \sqrt{\frac{3}{4}\frac{\nu'}{\nu}}}$$

It is a Lorentz invariant, i.e. the same in the laboratory and electron rest frame

$$E' = h\nu' = h\nu\gamma(1 + \beta\cos\theta) = E\gamma(1 + \beta\cos\theta)$$

Let's consider a region where there is a plasma of relativistic electrons and a radiation field:
in the electron framework, the photon energy is

$$\frac{d\varepsilon}{dt} = \frac{d\varepsilon'}{dt'} = c\sigma_T \frac{\langle E'^2_{rad}\rangle}{8\pi} = c\sigma_T\gamma^2\langle(1 + \beta\cos\theta)^2\rangle\langle E^2_{rad}\rangle$$

since the emitted energy is a Lorentz invariant. It is then possible to write

$$\langle E^2_{rad}\rangle = U_{ph} \qquad \langle(1 + \beta\cos\theta)^2\rangle = 1 + \frac{1}{3}\beta^2$$

and, finally, the energy coming out the scattering region is

$$\frac{d\varepsilon_{out}}{dt} = c\sigma_T U_{ph}\gamma^2\left(1 + \frac{1}{3}\beta^2\right)$$

The energy associated with the photons, prior of the scattering is

$$\frac{d\varepsilon_{in}}{dt} = -c\sigma_T U_{ph}$$

and then the net effect of the Inverse Compton scattering is

$$\frac{d\varepsilon_{out}}{dt} - \frac{d\varepsilon_{in}}{dt} = \left(\frac{d\varepsilon}{dt}\right)_{ic} = c\sigma_T U_{ph}\left[\gamma^2\left(1 + \frac{1}{3}\beta^2\right) - 1\right]$$

$\gamma^2 - 1 = \gamma^2\beta^2$ *then* $\left(\frac{d\varepsilon}{dt}\right)_{ic} = \frac{4}{3}c\sigma_T\gamma^2\beta^2 U_{ph}$

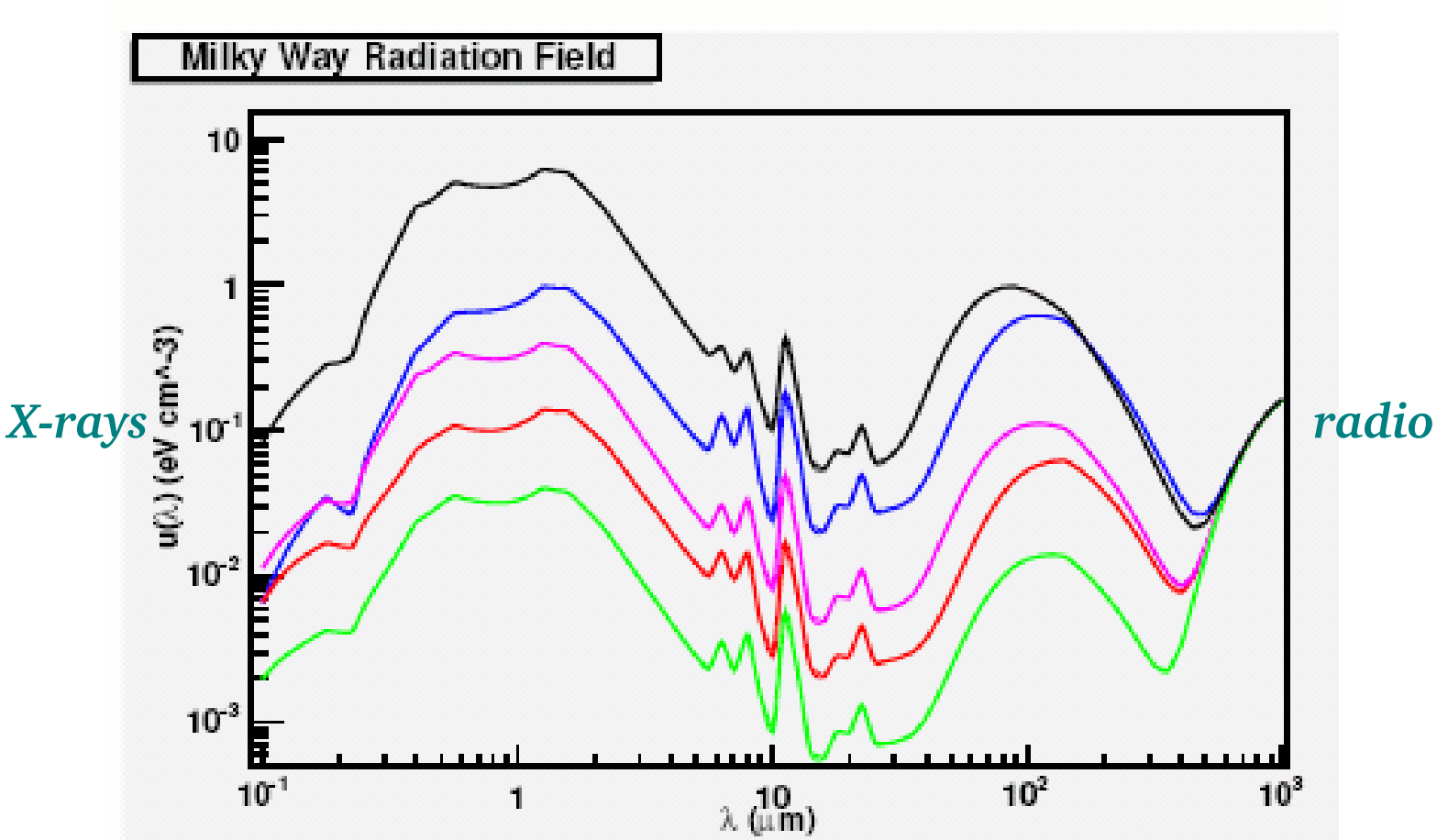
and if we recall the synchrotron emission $\left(\frac{d\varepsilon}{dt}\right)_{syn} = \frac{4}{3}c\sigma_T\gamma^2\beta^2 U_H$

we get

$$\frac{\left(\frac{d\varepsilon}{dt}\right)_{syn}}{\left(\frac{d\varepsilon}{dt}\right)_{ic}} = \frac{U_H}{U_{ph}}$$

The "Local Radiation Field"

Evaluation of U_{rad}: as taken in a number of selected locations



Spatial variation of the total radiation field throughout the Galaxy.

Black line *: total radiation field for GC.*

Magenta line *: total radiation field for R=0, z=5 kpc,*

Blue line *: total radiation field for R= 4, z=0 kpc,*

Red line *: total radiation field for R=12, z=0 kpc,*

Green line *: total radiation field for R=20, z=0 kpc.*

This diagram will be discussed a bit further when considering the ISM

Synchrotron & Inverse Compton scattering

The same relativistic electrons radiate via synchrotron and inverse Compton; their contributions add up:

$$-\left(\frac{d\varepsilon}{dt}\right)_{sy+ic} = C_s \varepsilon^2 \left[H^2 + 8\pi U_{rad}\right]$$

The increased cooling rate implies that the electron radiative lifetime is consequently reduced:

$$t^*_{sy+ic} \approx \frac{1}{\varepsilon^*} \frac{1}{H^2 + 8\pi U_{rad}} \approx \frac{3\cdot 10^8}{(H^2/8\pi + U_{rad})\ [eV\,cm^{-3}]} \frac{1}{\varepsilon^*\ [GeV]}\ yr$$

Therefore.... hard times for relativistic electrons!

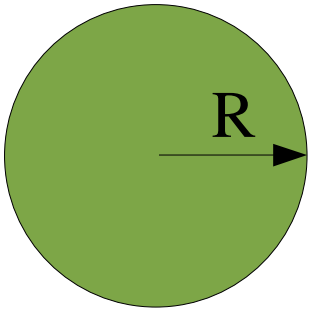
Example: radiative lifetime for a relativistic electrons in a radio lobe.

H_{eq}, U_{rad}, *strong dependence on z!*

$$B_{CMB} \sim 3.28\ (1+z)^2\ \mu G$$

Synchrotron Self-Compton Catastrophe

Let's consider a spherical cloud with radius R filled with magnetized relativistic plasma located at a distance d from the observer. The low-energy synchrotron photons may be up-scattered by the relativistic electrons via IC:



$$L_s = 4\pi d^2 \int S(\nu)\, d\nu \approx 4\pi d^2 S_{max} \nu_c f(\alpha) =$$

$$= U_{rad} \frac{4\pi c}{3} R^2$$

the energy density of the radiation field can then be derived

$$U_{rad} = \frac{3 L_s}{4\pi R^2 c}$$

finally the comparison between the IC and Synchrotron losses gives:

$$\frac{L_{IC}}{L_s} = \frac{U_{rad}}{H^2/8\pi} = \frac{6 L_s}{R^2 H^2 c} \approx \frac{S_{max}\nu_c}{\theta^2 H^2 c} f(\alpha) = \left(\frac{T_{Bmax}}{10^{12} K}\right)^5 \left(\frac{\nu_c}{GHz}\right) f(\alpha)$$

For brightness temperatures above 10^{12} K, the radiation field will undergo to a dramatic amplification; IC would become the most efficient and dominant cooling process (X-Rays) and a source would radiate its energy in a very short time [Compton catastrophe].

Brightness temperature limit in radio source is $\approx 10^{12}$ K

Comptonization:

We want to evaluate if there is a substantial

change in the spectrum of the seed photons in case of multiple scattering with hot (but non-relativistic) electrons in thermal equilibrium.

Let's consider the process from the point of view of the photons

There are interactions where energy is transferred from photons to electrons (C scattering)

$$\left\langle \frac{\Delta h\nu}{h\nu} \right\rangle_{phot} \approx -\frac{h\nu}{m_e c^2}$$

There are interactions where energy is transferred from electrons to photons (IC scattering)

(positive since we are considering the process from the point of view of the photons)

$$\left\langle \frac{\Delta \varepsilon}{\varepsilon} \right\rangle_{el} \approx \frac{4}{3}\left(\frac{v}{c}\right)^2 = \frac{4}{3c^2}\frac{3kT}{m_e} = \frac{4kT}{m_e c^2}$$

Thermal equilibrium
$(1/2)m_e v^2 = (3/2)kT$

The net energy transfer is:

$$\boxed{\left\langle \frac{\Delta E}{E} \right\rangle = \frac{(4kT - h\nu)}{m_e c^2}}$$

Comptonization: -2-

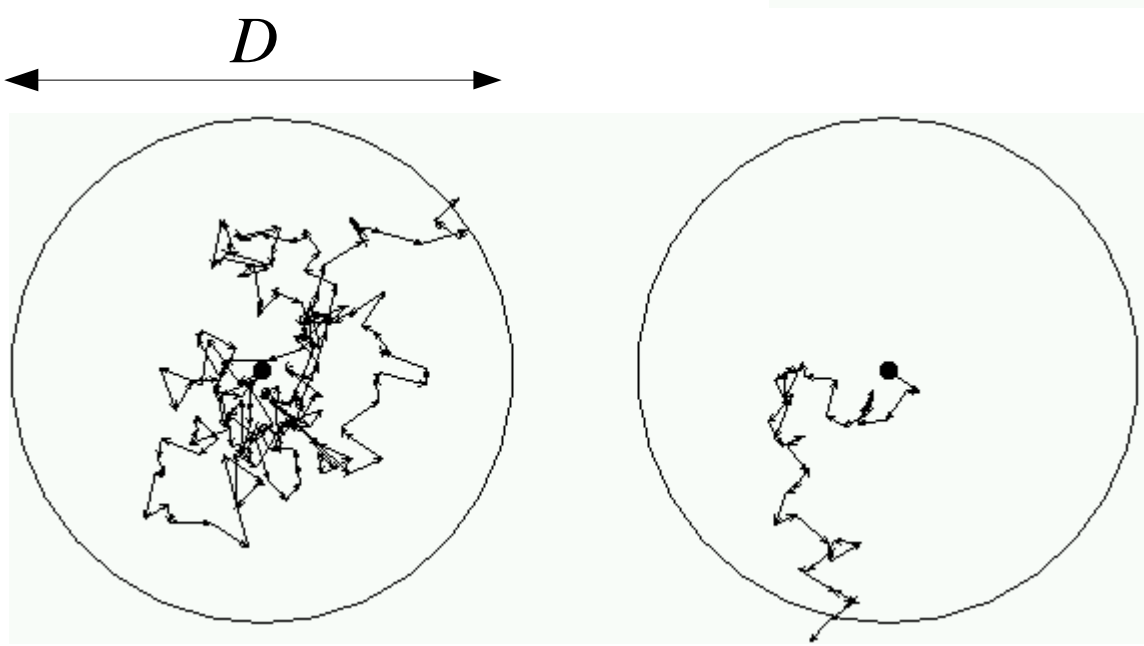
There is not a net energy transfer when $4kT = h\nu$

$4kT \ll h\nu$ photons loose energy and electrons (gas) heats up

$4kT \gg h\nu$ photons gain energy and electrons (gas) cools down

Let's consider the latter case in a region of size *D*:

$$< \frac{\Delta\mathcal{E}}{\mathcal{E}} > \simeq \frac{4kT}{m_e c^2}$$



The opacity to the scattering is

$$\tau_e = n_e \sigma_T D$$

and the mean free path is

$$l_{mfp} = \frac{1}{n_e \sigma_T}$$

Comptonization: -3-

The total number of collisions is:

$$N = \left(\frac{D}{l_{mfp}}\right)^2 = \tau_e^2$$

the total energy gain is

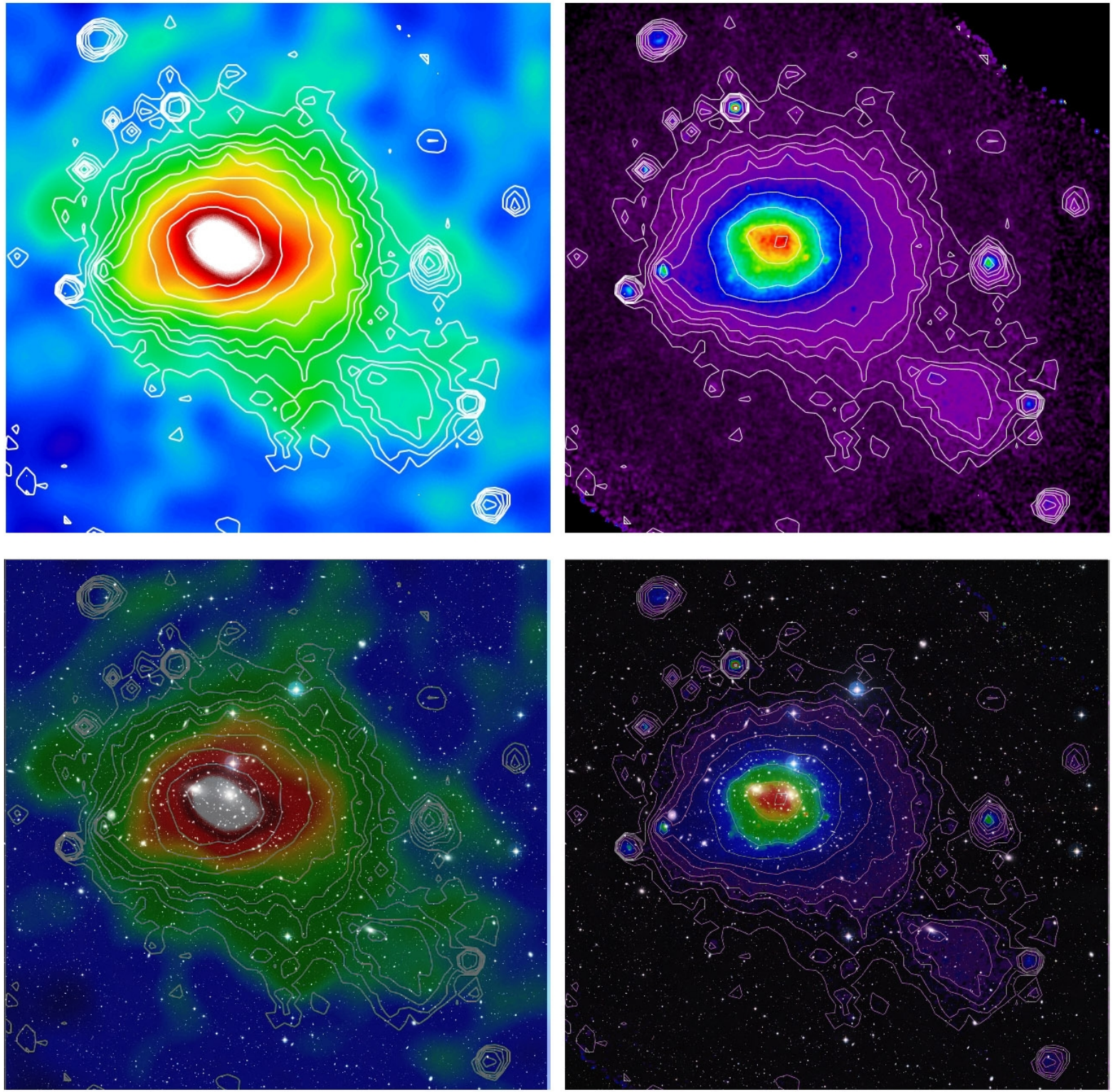
$$< \frac{\Delta\, h\nu}{h\nu} > \Big|_{tot} \approx \frac{4kT}{m_e c^2} N = \frac{4kT}{m_e c^2} \mathcal{N}\, dt$$

the mean photon energy as a function of time becomes

$$h\,\nu\,(t) = h\,\nu_o e^{(4kT/m_e c^2)\,\mathcal{N}\,t} = h\,\nu_o e^{(4kT/m_e c^2)\,N} = h\,\nu_o e^{4y}$$

where y is known as "Compton y-parameter"

$$y = \frac{kT}{m_e c^2} N = \frac{kT n_e^2 \sigma_T^2 D^2}{m_e c^2}$$



The spectrum of the radiation is substantially modified when $\nu_f >\sim \nu_o$

$$y \geq \frac{1}{4} \quad \text{i.e.} \quad \frac{4kT}{m_e c^2} N = \frac{4kT n_e^2 \sigma_T^2 D^2}{m_e c^2} \geq 1$$

and the final equilibrium is achieved when

$$h\nu_f = 4\mathrm{kT} \quad i.e. \quad y = \frac{1}{4} \ln\left(\frac{4\mathrm{kT}}{h\nu_o}\right)$$

namely

$$\tau_e = \left[\ln\left(\frac{4\mathrm{kT}}{h\nu_o}\right) \frac{m_e c^2}{4\mathrm{kT}} \right]^{1/2}$$

If this condition is met (thermal equilibrium), the modified photon spectrum must follow the Bose-Einstein distribution

Modified spectrum

At equilibrium, the photons will follow a Bose - Einstein distribution:

$$u(\nu)\,d\nu = \frac{8\pi h\nu^3}{c^3}\frac{1}{e^{(h\nu/kT)+\mu}-1}\,d\nu$$

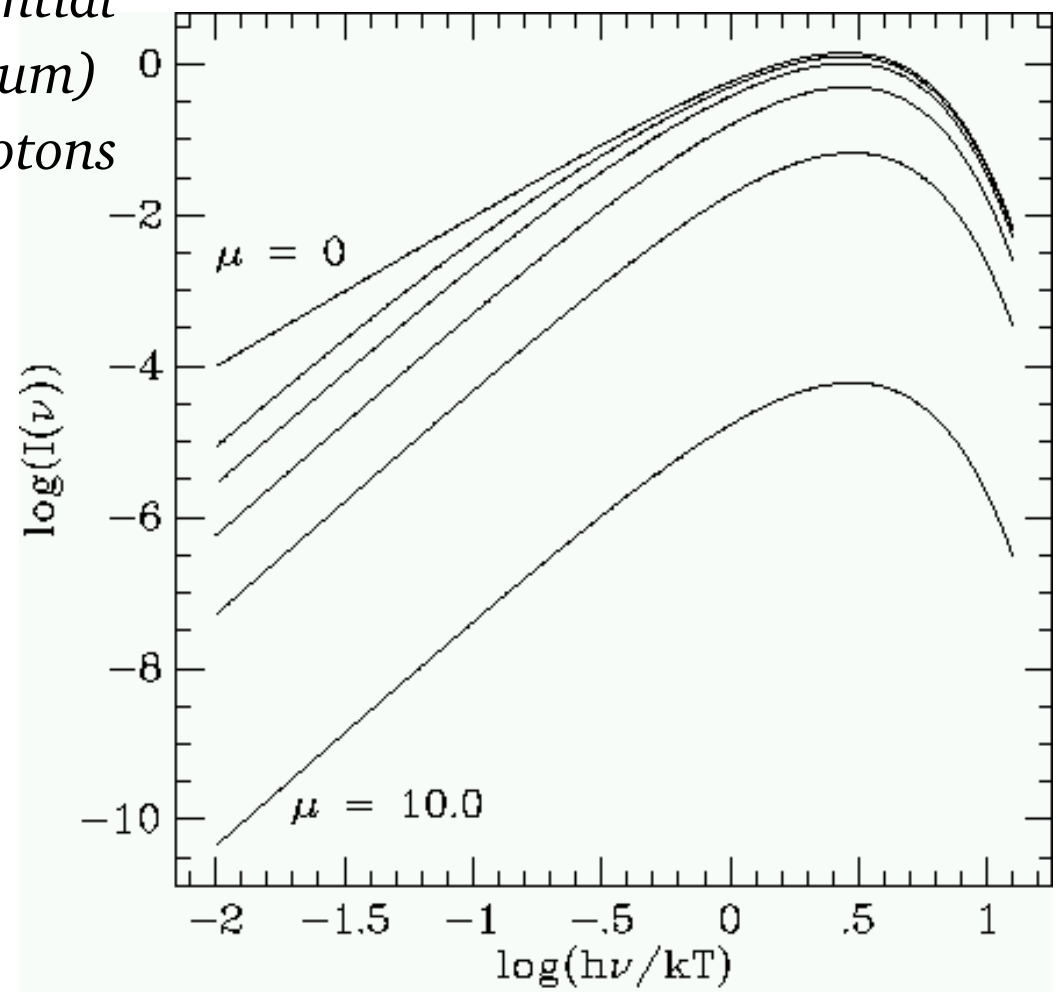
where μ *is known as chemical potential (which is =0 for Planck's BB spectrum) and measures the rate at which photons are produced*

in case $\frac{h\nu}{kT} + \mu \gg 1$

we have

$$u(\nu)\,d\nu = \frac{8\pi h\nu^3}{c^3} e^{-h\nu/kT}\, e^{-\mu}$$

i.e. the Wien's law modified by $e^{-\mu}$



Kompaneets' equation:

The exact description of the photon scattering (i.e. find the observed spectrum originated by various values of the y-parameter) is better obtained in the phase-space through the Kompaneets' equation:

$$\frac{\partial n}{\partial y} = \frac{1}{x^2}\frac{\partial}{\partial x}\left[x^4\left(n + n^2 + \frac{\partial n}{\partial x}\right)\right]$$

where

$$n = \frac{u(\nu)c^3}{8\pi\, h\nu^3} \qquad x = \frac{h\nu}{kT}$$

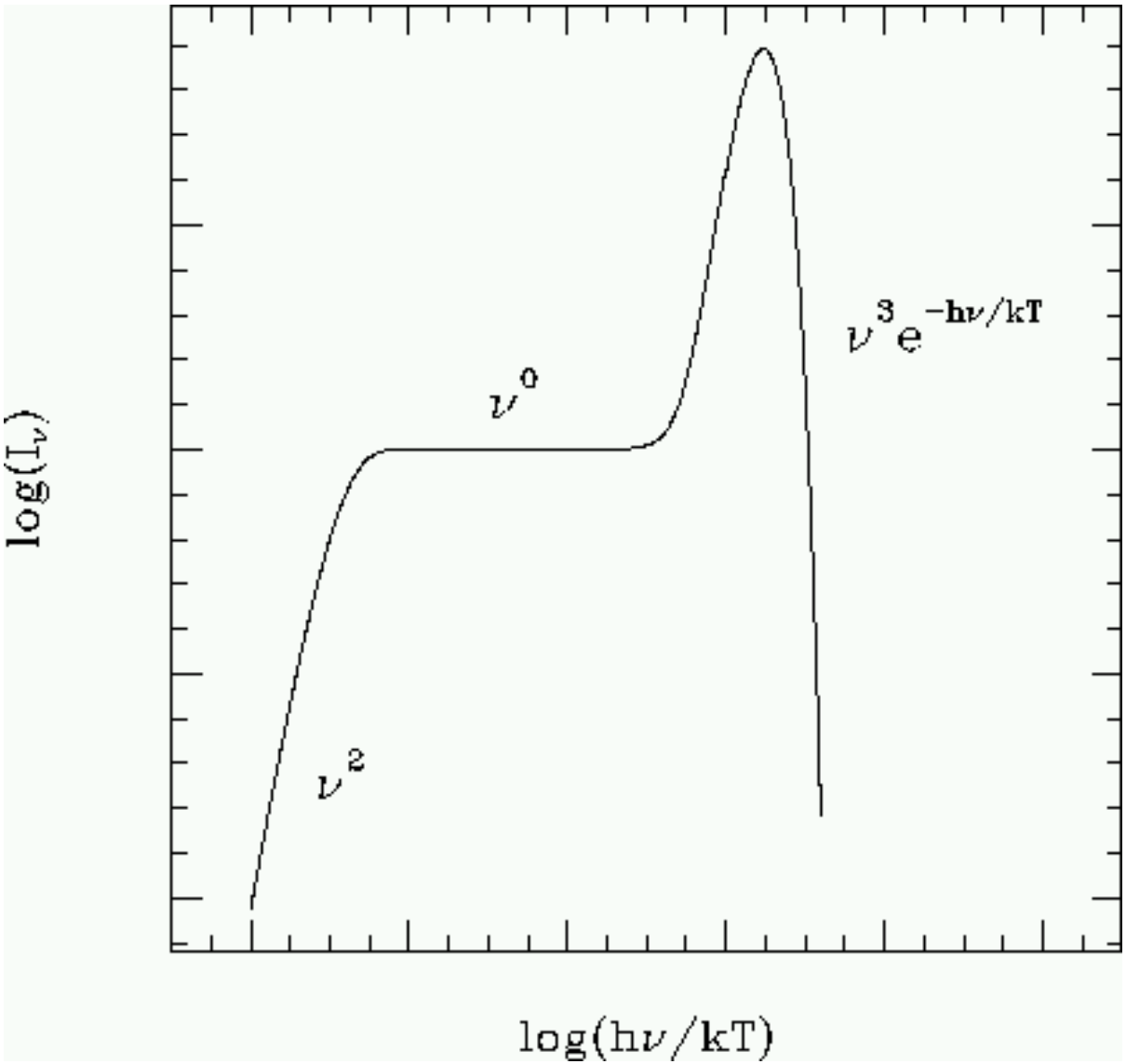
Recoil effect

Induced/stimulated emission

Doppler motion

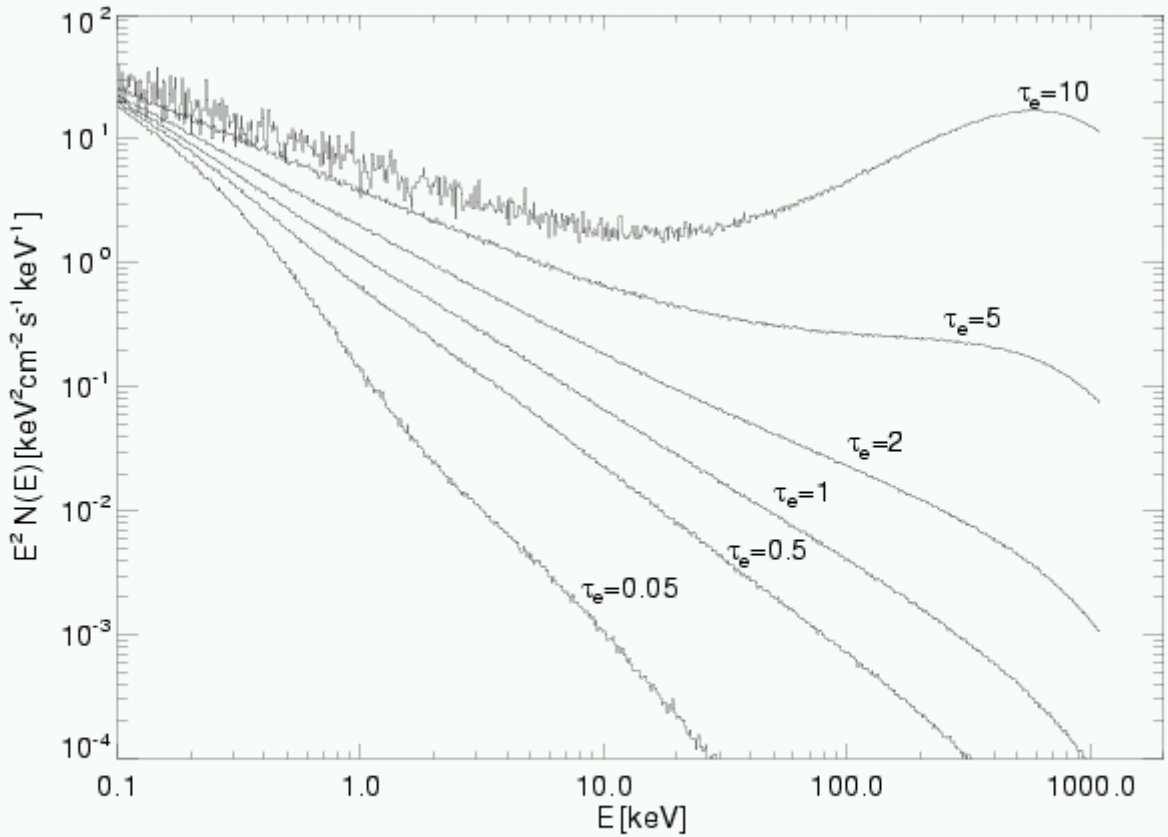
Emission spectra come out as power-law with index 3+m where $m = -\frac{3}{2} \pm \sqrt{\frac{9}{4} + \frac{1}{y}}$

Examples: ***Accretion disks in AGNs , and μQ;*** ***S-Z effect***



Thermal bremsstrahlung spectrum with comptonization
(accretion disks)

Comptonization: *the spectrum is modified as a function of the opacity to this process*



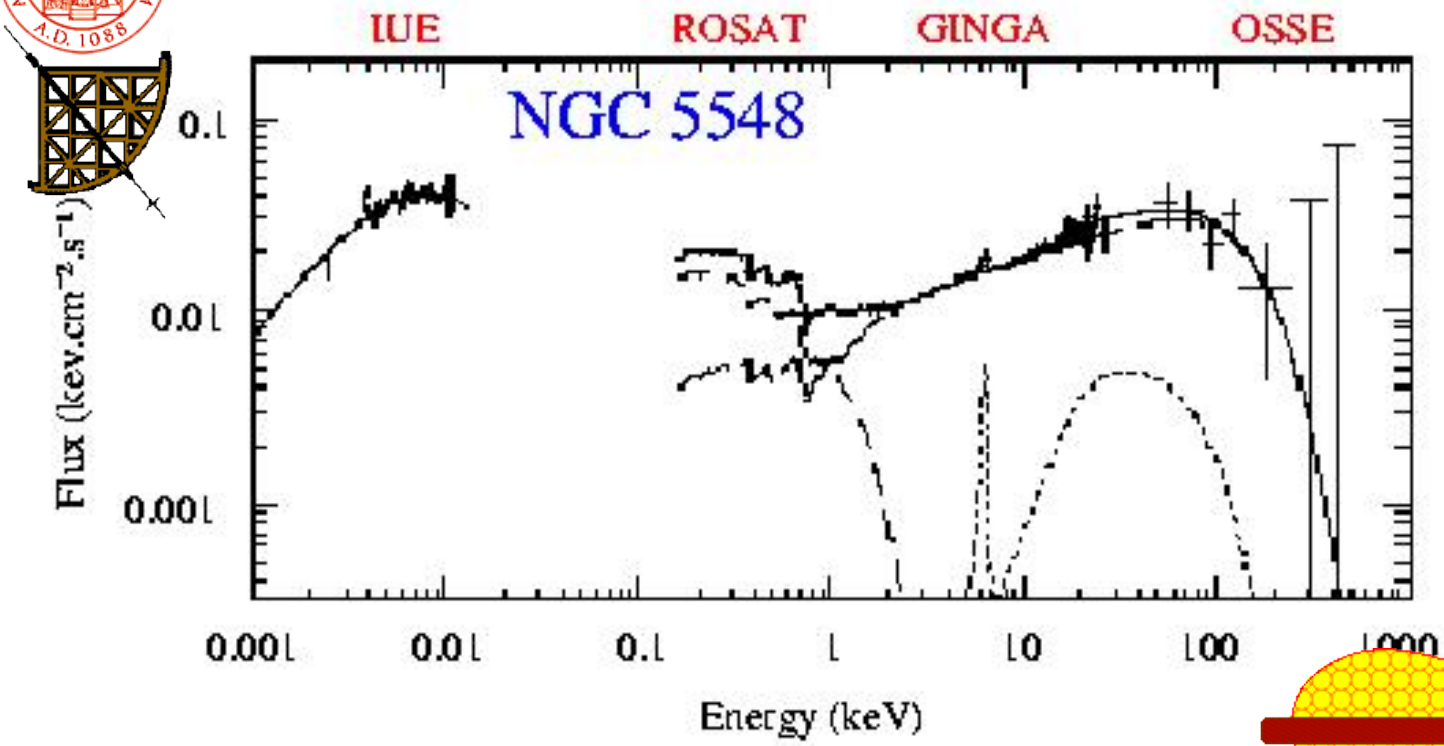
Sphere with $kT_e = 0.7 m_e c^2$ ($\sim$ 360 keV), seed photons come from center of sphere.

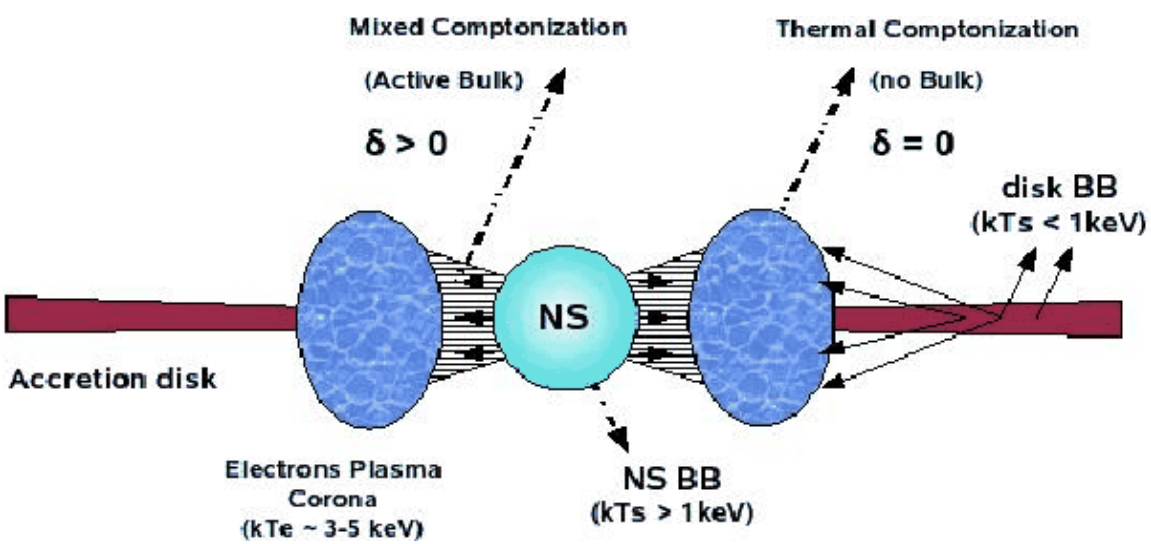
$y \ll 1$: pure power-law spectrum.

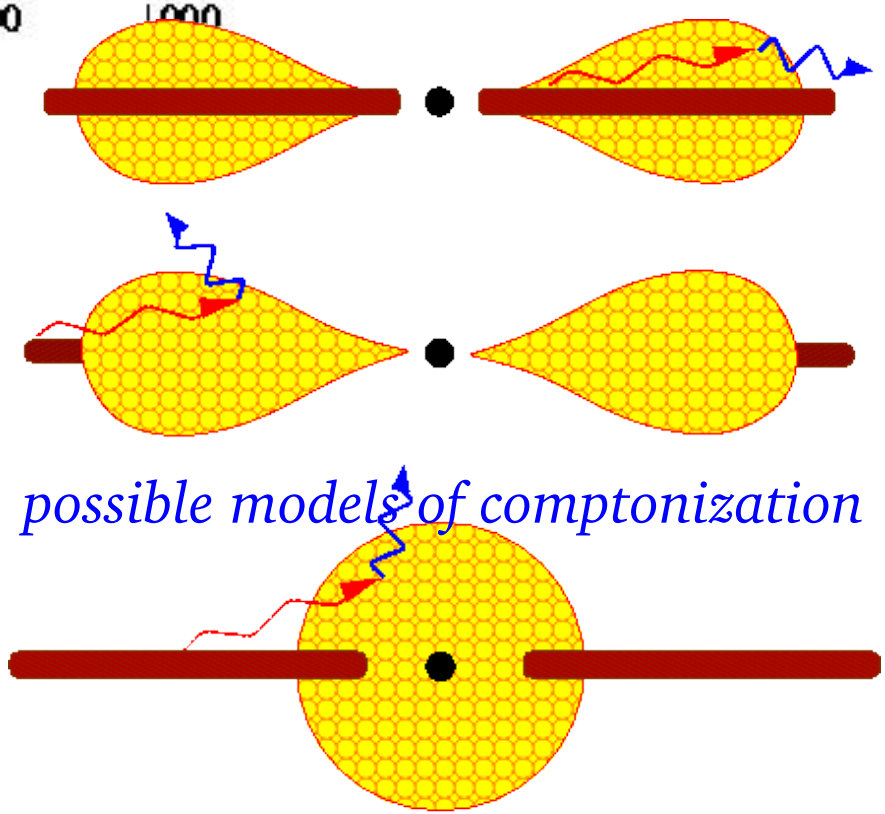
$y < 1$: power-law with exp. cut-off.

$y \gg 1$: "Saturated Comptonization".

Comptonization: *Astrophysical examples*

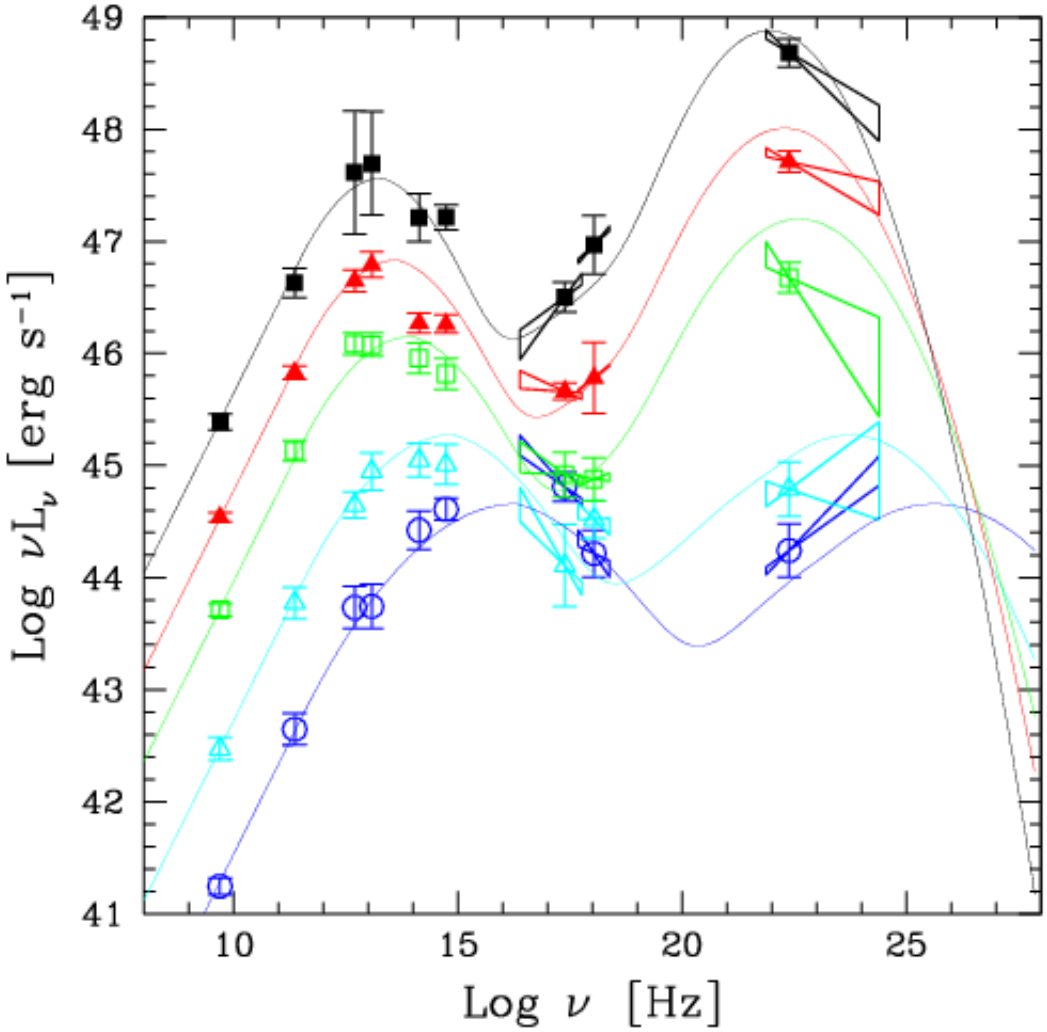






possible models of comptonization

Comptonization: Astrophysical examples (2)



courtesy of G. Ghisellini

Sunyaev-Zeldovich effect

- *thermal: high energy electrons interact with CMB photons via IC scattering*

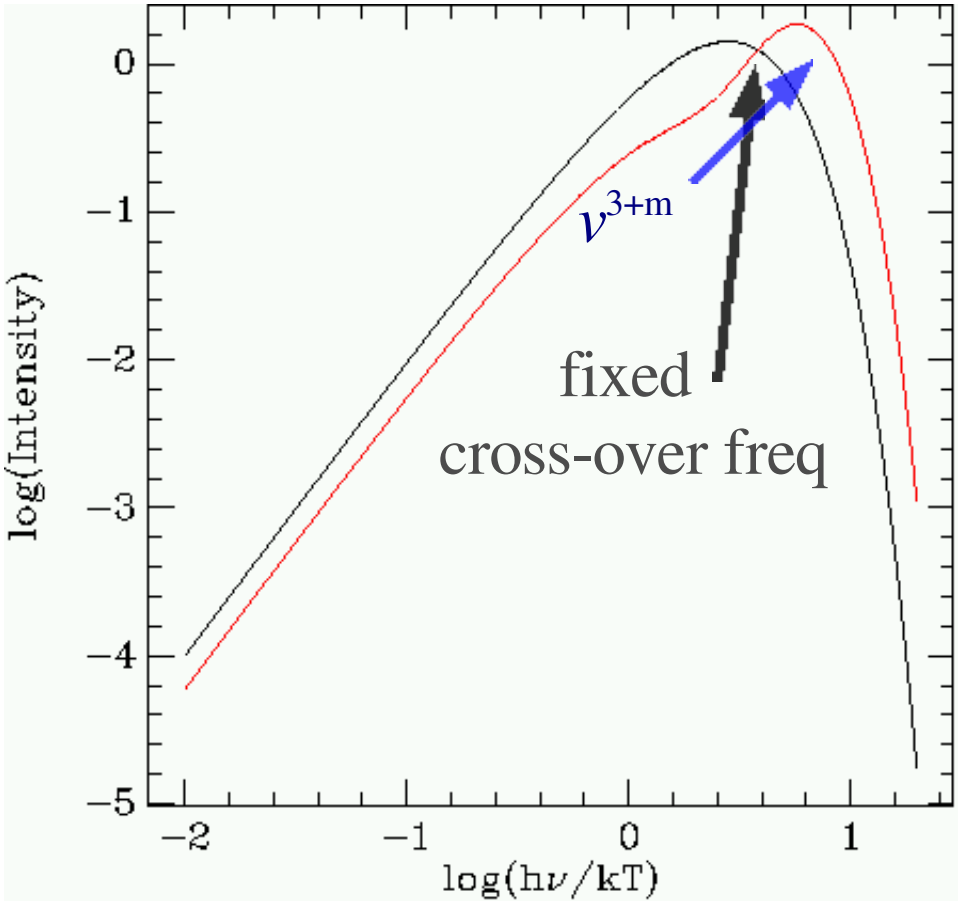
- *kinematic: bulk motions modify the spectrum, second order effect, also known as Ostriker–Vishniac effect*

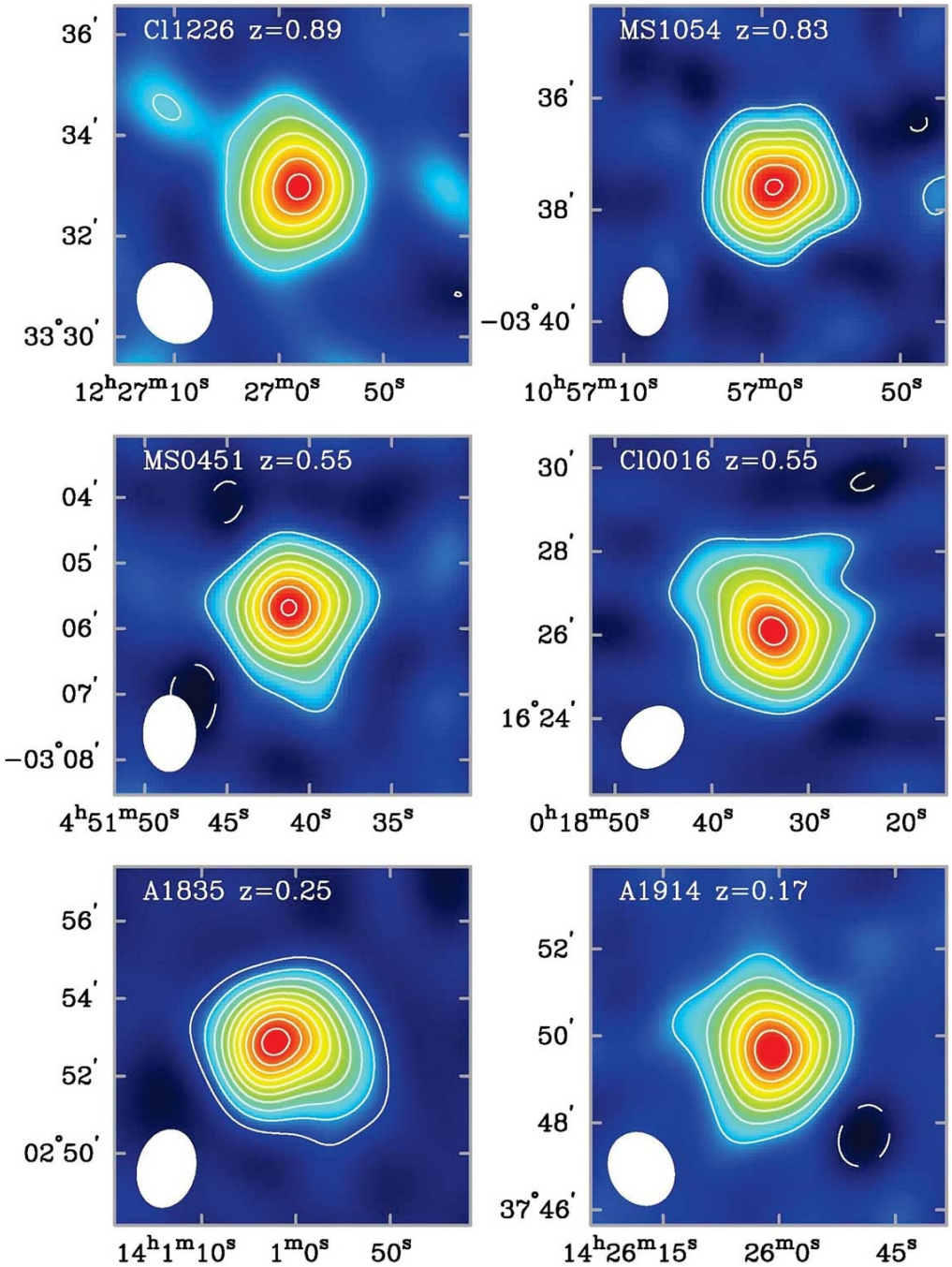
Where does it take place?

Sunyaev-Zeldovich effect

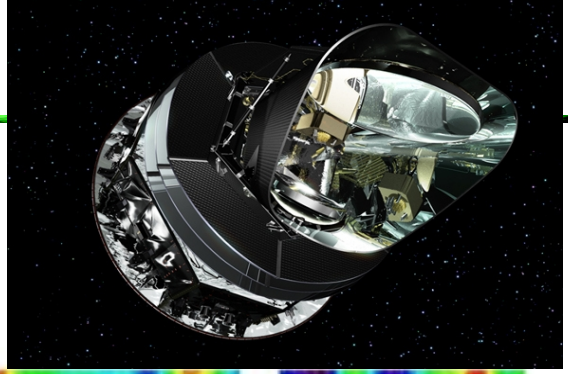
Galaxy clusters have hot electrons capable of IC scattering of CMB photons.

A CMB photon have a 1% probability of interacting with a high energy ICM electron.





Sunyaev-Zeldovich effect: the Planck's view



The same galaxy cluster as a function of frequency as seen by the Planck satellite

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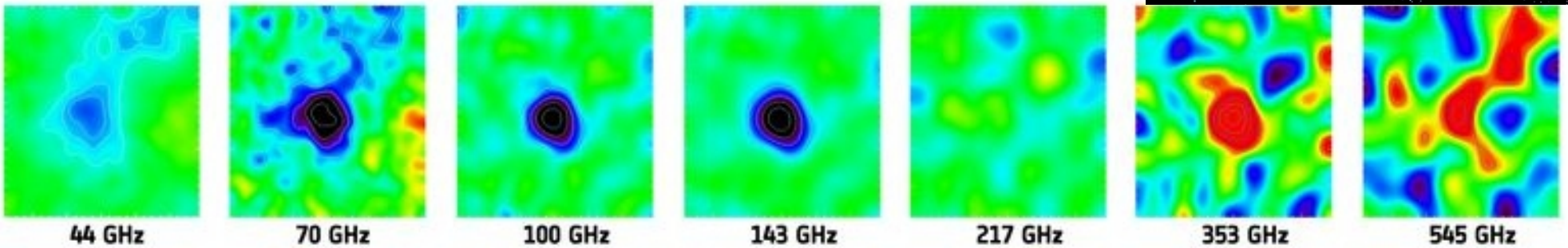

A full sky temperature map is at
http://www.astro.cardiff.ac.uk/~spxcen/CMB_Sims/Planck_comb_rbcol_scaled.png