Acceleration mechanisms

Stochastic: - collisions among particles/clouds

Systematic: - H field compression + scattering/diffusion
- shocks
- e-m processes (e.g. Low frequency-large amplitude waves in pulsar magnetosphere)
Fermi's Acceleration:

Proposed by E. Fermi in 1954.

Let's consider a charged particle, given that electrons are known to achieve very high energies (e.g. cosmic rays, synchrotron radiation).

Electrostatic fields cannot survive given the enormous conducibility. There is a magnetic field in the cloud only. However, when the charge moves into the moving cloud (seen from the observer's frame) it "feels" an electric field as well.

\[ F'_e = e \mathbf{E}' = e \left[ \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{H} \right] = m_e \frac{d\mathbf{v}_e}{dt} \]

\[ \mathbf{E} \approx -\frac{\mathbf{u}}{c} \times \mathbf{H} \]

is the field produced by the moving cloud, which is moving at a speed \( \mathbf{u} \) while the electron moves at \( \mathbf{v} \)

\[ m_e \frac{d\mathbf{v}_e}{dt} = e \left[ -\frac{\mathbf{u}}{c} \times \mathbf{H} + \frac{\mathbf{v}}{c} \times \mathbf{H} \right] \]
After a scalar product with the charge velocity we get

\[
   m_e \frac{dv_e}{dt} = e \left[ - \frac{u}{c} \times H + \frac{v}{c} \times H \right]
\]

\[
   \frac{d}{dt} \left( \frac{1}{2} m_e v^2 \right) \approx \frac{e}{c} v \cdot (u \times H)
\]

(in fact the second term is zero!)

This means that the energy of the electron change in case the Lorentz force is active, and this requires that the magnetized cloud is in motion.
Fermi's Acceleration:

Elastic collision between a cloud and a particle with $|v| \gg |u|$: conservation of $E$ and $p$

$$v' = \frac{(m - M)v \pm 2Mu}{m + M} \approx -v \pm 2u$$

$$u' \approx u$$

In terms of particle energy $(\epsilon, \epsilon')$

$$\epsilon' = \frac{1}{2} m (v')^2 = \frac{1}{2} m (-v \pm 2u)^2$$

$$= \frac{1}{2} m (v^2 \pm 4uv + 4u^2) = \epsilon (1 \pm \frac{4u}{v} + 4\frac{u^2}{v^2})$$

$$\Delta \epsilon = \frac{4u}{v} \epsilon$$ \hspace{1cm} Type I collision

$$\Delta \epsilon = -\frac{4u}{v} \epsilon$$ \hspace{1cm} Type II collision
Fermi's Acceleration:

Type I interactions happen more often than Type II

\[ f_I = \frac{v+u}{l} \quad f_{II} = \frac{v-u}{l} \]

\[ \langle \frac{\Delta \varepsilon}{\Delta t} \rangle_F = f_I \Delta \varepsilon_I + f_{II} \Delta \varepsilon_{II} = 4 \frac{u}{v} \varepsilon \frac{2u}{l} = \frac{8v}{l} \left( \frac{u}{v} \right)^2 \varepsilon = \frac{\varepsilon}{\tau_F} \]

A factor 2 is more appropriate than 8 (valid for head on collisions only)

If we integrate in time we obtain:

\[ \varepsilon(t) = \varepsilon_o e^{\frac{t}{\tau_F}} \quad \tau_F \approx \frac{lv}{2u^2} \]

once the particle is accelerated at the required velocity, then should be able to leave the region where acceleration takes place. Namely the confining time \( \tau_c \) should be of the order of (or slightly larger than) the acceleration time \( \tau_F \)

For initial velocities of \( \sim 10 \) km/s and the cloud size and number density (distance 10-100 pc) relativistic velocities are achieved at

\[ \tau_F \approx 10^{11} \text{ yr} \]
Fermi's Acceleration:

In SNR, “clouds” have higher velocities (thousands km/s) and $l$ is small (0.1 pc) and

$$\tau_F \approx 10^5 \text{ yr}$$

Spectrum of Fermi's acceleration processes:

- $k = \# \text{ of collisions}$
- $\beta = \text{energy increase per collision}$
- $p = \text{probability to remain within the acceleration region}$

$$\epsilon_k = \epsilon_o \beta^k$$

$$N_k = N_o \ p^k$$

$$\frac{\ln(N_k/N_o)}{\ln(\epsilon_k/\epsilon_o)} = \frac{\ln(p)}{\ln(\beta)} = m$$

$$N_k = N_o \left( \frac{\epsilon_k}{\epsilon_o} \right)^m \rightarrow N(\epsilon) \, d\epsilon = \text{cost} \ \epsilon^{-1+m} \, d\epsilon$$

i.e. power–law energy distribution
Observations of radio supernovae

The observations shown aside (images on the same scale!) require that efficient particle acceleration takes place on time scales as short as a few weeks.
Shock waves and Fermi's collisions (1)

\[ c_s = \sqrt{\frac{\gamma k T}{\mu m_H}} \]

sound speed for an ideal gas (no H field)

if a perturbation moves at a speed exceeding \( c_s \), a discontinuity is created

region of particles to be accelerated is moving at \( v_2 \)

strong shock is moving at \( v_1 \)

if \( v_2 = (3/4)v_1 \) particles cross several times the shock front before they gain enough energy to leave the acceleration region
Shock waves and Fermi's collisions (2)

The combination of the two velocities allow a particle to have Fermi – I type collisions in a row (and rebounds with unperturbed clouds at rest).

\[ v_2 = (3/4) v_1 \quad \Delta \varepsilon_1 = 0 \]

\[ \Delta \varepsilon_2 \approx 2 v_2 \frac{\varepsilon}{v} = \frac{3}{2} \frac{v_1}{v} \varepsilon \]

The occurrence between collisions is

\[ f \approx \frac{v}{2l} \]

And the energy gain in time is

\[ \frac{d\varepsilon}{dt} \approx \frac{3}{2} \frac{v_1}{v} \varepsilon \frac{v}{2l} \approx \left( \frac{3 v_1}{4 l} \right) \varepsilon = \frac{\varepsilon}{\tau_F} \]

\[ \tau_c \approx \frac{l}{v_1} \quad \text{confining time} \]

\[ \delta = \left(1 + \frac{\tau_F}{\tau_c}\right) \]

\[ \text{unperturbed } v = 0 \]
Shock waves and Fermi's collisions (3): an example. Cas A

2720 Jy at 1 GHz. The SN occurred at a distance of approximately 11,000 ly away. The expanding cloud of material left over from the supernova is now approximately 10 ly across. Despite its radio brilliance, however, it is extremely faint optically, and is only visible on long-exposure photographs. It is believed that first light from the stellar explosion reached Earth approximately 300 years ago but there are no historical records of any sightings of the progenitor supernova, probably due to interstellar dust absorbing optical wavelength radiation before it reached Earth (although it is possible that it was recorded as a sixthmagnitude star by John Flamsteed on August 16, 1680).

It is known that the expansion shell has a temperature of around 50 million degrees Fahrenheit (30 megakelvins), and is travelling at more than ten million miles per hour (4 Mm/s).

A false color image compositied of data from three sources. Red is infrared data from the Spitzer Space Telescope, orange is visible data from the Hubble SpaceT, and blue and green are data from the Chandra X-ray Observatory.
Accelerazione di Betatrone:

Sites like “magnetic traps” there is transfer of energy $\perp \Leftrightarrow \parallel$

$H$ is compressed $\parallel \Rightarrow \perp$

$H$ expands $\perp \Rightarrow \parallel$

*no net energy change,*

unless..... random interactions redistribute the energy increase at $(H+\Delta H)$ to other particles
\[
\begin{align*}
P^2 \parallel &= \frac{1}{3} P^2_1 \\
P^2 \perp &= \frac{2}{3} P^2_1
\end{align*}
\]

At location (1+)

\[
\begin{align*}
P^2 \parallel &= \frac{1}{3} P^2_1 \\
P^2 \perp &= \frac{2}{3} P^2_1 \left( \frac{H + \Delta H}{H} \right)
\end{align*}
\]

At location (2-)

\[
P^2_{tot} = P^2_2 = \frac{1}{3} P^2_1 \left( 1 + 2 \frac{H + \Delta H}{H} \right)
\]
\[
\begin{align*}
\begin{cases}
  P_{\parallel}^2 = \frac{1}{3} P_2^2 = \frac{1}{9} P_1^2 \left(1 + 2 \frac{H + \Delta H}{H}\right) \\
  P_{\perp}^2 = \frac{2}{3} P_2^2 = \frac{2}{9} P_1^2 \left(1 + 2 \frac{H + \Delta H}{H}\right)
\end{cases}
\end{align*}
\]

\[P_{tot}^2 = P_2^2 = \frac{1}{3} P_1^2 \left(1 + 2 \frac{H + \Delta H}{H}\right)\]

At location (2+)

\[
\begin{align*}
\begin{cases}
  P_{\parallel}^2 = \frac{1}{3} P_2^2 = \frac{1}{9} P_1^2 \left(1 + 2 \frac{H + \Delta H}{H}\right) \\
  P_{\perp}^2 = \frac{2}{3} P_2^2 = \frac{2}{9} P_1^2 \left(1 + 2 \frac{H + \Delta H}{H}\right) \frac{H}{H + \Delta H}
\end{cases}
\end{align*}
\]

\[P_{tot}^2 = P_3^2 = \frac{1}{9} P_1^2 \left(1 + 2 \frac{H + \Delta H}{H}\right) \left(1 + 2 \frac{H}{H + \Delta H}\right)\]

At location (3-)
In 3 again the particle is at the same initial $H$ field (as in 1) and the total momentum change is

$$\Delta p^2 = p_3^2 - p_1^2 = \frac{2}{9} p_1^2 \frac{H}{H + \Delta H} \left( \frac{\Delta H}{H} \right)^2 \approx \frac{2}{9} p_1^2 \left( \frac{\Delta H}{H} \right)^2$$

and in terms of energy:

$$\varepsilon = p^2/2m$$

$$\Delta \varepsilon \approx \frac{2}{9} \varepsilon \frac{\Delta H}{H}$$

$$\frac{d\varepsilon}{dt} \approx \frac{\Delta \varepsilon}{\Delta t_H} \approx \frac{2}{9} \frac{d(\ln(H))}{dt} \varepsilon$$
time for CRs
Let's consider an astrophysical plasma, composed by ionized gas which is, however, neutral as a whole. Maxwell equations are defined for vacuum, but can be adapted to a plasma if we consider charge and current densities $\rho_e$ and $j$

Let's consider the dielectric constant:

$$\epsilon_r = 1 - \frac{4\pi e^2}{m_e} \left( \frac{n_e}{\omega^2 - \omega_o^2} + \sum_i \frac{N_i}{\omega^2 - \omega_i^2} \right)$$

In the radio domain, $\omega < \omega_i$ and can be neglected.
We define the refraction index:

\[ n_r \equiv \sqrt{\varepsilon_r} \approx \sqrt{1 - \frac{4\pi e^2}{m_e} \frac{n_e}{\omega^2}} = \sqrt{1 - \left(\frac{v_p}{v}\right)^2} \]

where the plasma frequency \( v_p \) has been defined

\[ v_p = \sqrt{\frac{e^2 n_e}{\pi m_e}} = 9.1 \cdot 10^3 \sqrt{n_e} \quad (\text{Hz}) \]

only waves with \( v > v_p \) can travel across the region, while those with \( v < v_p \) are reflected (\( n_r \) becomes imaginary)

Below the plasma cutoff frequency there is no propagation

In the ionosphere: \( n_e \sim 10^6 \text{ cm}^{-3} \) implies \( v_p \sim 10^7 \text{ Hz} \)

In the interstellar medium: \( n_e \sim 10^{-3} 10^3 \text{ cm}^{-3} \)
\[ \text{implies } v_p \sim 3 \cdot 10^2 3 \cdot 10^3 \text{ Hz} \]
Wave propagation

\[ \nu > \nu_p \implies \text{the e-m wave travels with group velocity} \]

\[ v_g \approx \frac{\partial \omega}{\partial k} = c \cdot n_r = c \sqrt{1 - \left(\frac{\nu_p}{\nu}\right)^2} \approx c \left[1 - \frac{1}{2} \left(\frac{\nu_p}{\nu}\right)^2\right] \quad \text{(for } \nu >> \nu_p) \]

The time necessary to travel from A to B at a given frequency is

\[ T_{A,B}(\nu) = \int_0^L \frac{dl}{v_g} \approx \frac{1}{c} \int_0^L \left[1 - \frac{1}{2} \left(\frac{\nu_p}{\nu}\right)^2\right]^{-1} dl = \frac{1}{c} \int_0^L \left[1 + \frac{1}{2} \left(\frac{\nu_p}{\nu}\right)^2\right] dl = \frac{1}{c} \int_0^L dl + \frac{1}{2c} \int_0^L \left(\frac{\nu_p}{\nu}\right)^2 dl \]

\[ T_{A,B}(\nu) \approx \frac{L}{c} + \frac{1}{2c} \left[\frac{e^2}{\pi n_e \nu^2}\right] \int_0^L n_e dl \quad \text{D.M.} = \int_0^L n_e dl \]
In case it is possible to detect this effect in a given particular case, then it becomes feasible to directly measure $n_e$ along the LOS.
Dispersion Measure

The slope of the pulse arrival time vs. frequency provides a measure of

$$D.M. = \int_0^L n_e \, dl$$

Distances, however, are difficult to determine, except in a few lucky cases, like globular clusters.
Faraday Rotation

Propagation effect arising from an “external” magnetic field $H$ which causes an anisotropic transmission. Let’s consider what happens along the field direction:

$$m \frac{d \mathbf{v}}{dt} = -e \left( \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{H} \right)$$

$$\omega_H = \frac{eH}{mc}$$

Let’s us assume that the propagating e-m wave is polarized and sinusoidal as a superposition between a LCP and a RCP components.

$$\vec{E}(t) = E_o e^{-i\omega t} (\vec{e}_1 \pm \vec{e}_2)$$

where $+$ is for RCP and $-$ is for LCP. The dielectric constant is no longer a scalar and becomes a tensor: the “two” modes have different refraction index

$$(n_r)_{R,L} = \sqrt{1 - \left( \frac{\nu_p}{\nu} \right)^2 \frac{1}{1 \pm (\nu_p / \nu) \cos \theta}}$$

where $\theta$ is the angle between the direction of e-m wave propagation and $\vec{H}$.
Faraday Rotation

Along the field direction

$$\vec{B}_o = B_o \vec{e}_3$$

and then in equation ** we get as a solution

$$\vec{v}(t) = -\frac{ie}{m_e(\omega \pm \omega_H)} \vec{E}(t)$$

which provides a dielectric constant

$$\varepsilon_{R,L} = 1 - \frac{\omega_p^2}{\omega(\omega \pm \omega_H)}$$

and therefore the propagation speed of the two orthogonal modes are different, originating a shift in their relative phase, which implies a rotation of the polarization vector
How to measure RM:

Polarization sensitive observations provide the measurement of $m$ and $\chi$ at various discrete frequencies (wavelengths).

- Plot $\lambda^2$ and $\chi$ ($\pm n\pi$)
- Get the slope of the best fit line
- Obtain RM

Blue points represent the observations, while red points are for the $\pm n\pi$ ambiguities.
In case the “Faraday screen” is spatially resolved, the net effect is just a rotation of the linear polarization vector in a given direction.

Over a distance $D$ there is a phase $\phi_{RL}$ depending on the wave number

$$
\phi_{RL} = \int_0^D k_{R,L} dl = \int_0^D (k_R - k_L) dl = \int_0^D \frac{\omega}{c} \left[ \sqrt{1 - \frac{\omega_p}{2\omega^2}} \left( 1 + \frac{\omega_H}{\omega} \right) - \sqrt{1 - \frac{\omega_p}{2\omega^2}} \left( 1 - \frac{\omega_H}{\omega} \right) \right] dl
$$

$$
\Delta \theta = \frac{1}{2} \phi_{RL} = \frac{2\pi e^3}{m^2 c^2 \omega^2} \int_0^D n_e H || dl \approx \chi^2 \int_0^D n_e H || dl \quad \text{(radians)}
$$

$$
R.M. = \frac{2\pi e^3}{m^2 c^2} \int_0^D n_e H || dl
$$

The rotation measure determines the magnetic field along the LOS weighted on the electron density $n_e$ and if also the D.M. is available

$$
\langle H || \rangle \propto \frac{R.M.}{D.M.} \propto \frac{\int n_e H || dl}{\int n_e dl}
$$
Examples of RM and FR

RM needs various frequencies to be measured

Depolarization may take place

RM may be very different in various locations of the same radio source -> local to the r-source
Examples of RM and FR (2)

Stratified RM structure (implications on $H$ and $n_e$)