Some Notes on Beamforming.


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1) Introduction: consideration on beamforming

Beamforming is an alternative name for spatial filtering where, with appropriate analog or digital signal processing, an array of antennas, can be steered in a way to block the reception of radio signals coming from specified directions. While a filter in the time domain combines energy over time, the beamformer combines energy over its aperture, obtaining a certain antenna gain in a given direction while having attenuation in others. Beamforming has been used for many years in different radio applications such as communications, surveillance, radar and, with different array sensors, in sonar and audio fields. The traditional analog way to perform beamforming was very expensive, and it was sensitive to component tolerances and drifts, while modern technology offers high speed A/D converters and Digital Down Converters (DDCs), fundamental blocks for digital beamforming. In both analog and digital domains the most common methods used to create directional beams are the time delay (time shift) and phase shift ones. The time delay approach allows to form and steer the beam by adding adjustable time delay steps that are independent from the operating frequency and bandwidth. Since it is difficult to generate time delays in both the analog and digital domains, they are used only when strictly requested as, for instance, with large arrays and/or when the bandwidth of the system is wide. In the case of phase shifting, a phase is introduced instead of applying true time delays for each receiver. It is simple to introduce such a compensation but unfortunately this works properly only with narrow band systems and/or small arrays. In practice the use of time or phase shift is determined by the loss of gain that can be accepted. The normalised gain depends on the bandwidth B and the delay Δt (different time of arrival of the front wave at the antenna elements due to the physical dimension of the array) as reported in the following expression (normalised to 1):

\[
G = \frac{\sin(\pi B \Delta t)}{\pi B \Delta t}
\]

The linear equally spaced (LES) array has to be considered the most suitable configuration for a basic beamforming implementation. It can be easily noted that a LES is a spatial domain form of a temporal Finite Impulse Response (FIR) filter. As shown in Fig. 1.1, the wave plane arrives at each of the antenna elements 1, 2, 3…N (here N=8) at different times, depending on the incidence angle α. In the beamforming phase, the individual delays have to be adjusted to make the signal arrive simultaneously on each antenna from angle α [1]. The required time delay for every antenna of the array is given by:

\[
\Delta t = \frac{(n - 1) d \sin \alpha}{c}
\]

where “c” is the speed of light and “d” the element spacing.
Introducing such a delay on each element, the array can be steered in a specified direction while the use of amplitude weights $W_n$ allow to control the sidelobe levels and the nulls steering for radio interference rejection. A beamformer of $N$ antenna elements is able to steer the array on a desired direction and rejects (null beam steering) at maximum $N-1$ interfering signals coming from $N-1$ different directions. As already seen, this leads to consider the array as a FIR filter in the spatial domain, then the weights can be computed similarly to the coefficients of a standard FIR filter. Taking the FFT of the amplitude coefficients $W_n$, the radiation pattern $S(\alpha)$ of the array obtained with the beamformer, is given by the expression:

$$S(\alpha) = \sum_{n=0}^{N-1} W_n e^{j(\xi n d \sin \alpha)}$$  \hspace{1cm} (3)

where $\xi = 2\pi/\lambda$ (rad/m) and $\lambda$ (m) is the wavelength. Using the same weights for all the elements of the array ($W_n=1$), the resulting sidelobes are at -13 dB, while optimising the weights, the sidelobe levels can be drastically reduced to -20/-25 dB.

In the block diagram of a basic beamformer, shown in Fig. 1.2, we can consider the signal arriving at the antenna 1 to have a phase=0, while the signal that arrives at the antenna “$n$” leads in phase with $\xi n d \sin \alpha$. This leads to the definition of the **Array Propagation Vector** that contains the information of the direction of arrival of the signals:

$$\nu = \begin{bmatrix} e^{j\xi d \sin \alpha} & \ldots & e^{j(N-1)\xi d \sin \alpha} \end{bmatrix}$$  \hspace{1cm} (4)

In this way the **Array Factor** can be defined as:

$$F(\xi, \alpha) = W^T \nu$$  \hspace{1cm} (5)
Considering a fixed values for $\xi$ and $d$, the *Array Factor* represents the pattern $S(\alpha)$.

As an application example, we consider a weight factor defined by:

$$W_n = e^{jna} \quad (6)$$

If the wave front arrives to the array antennas from the angle $\phi_0$, in order to maximise the response in that direction, the phase of the complex weight has to be

$$\alpha = -\xi d \sin \phi_0 \quad (7)$$

In Fig. 1.3 a basic block diagram of a possible digital beamforming concept is visible. Signals from the receivers are down converted to the IF frequency and then digitised at a proper Nyquist frequency. A Digital Down Converter (DDC) produces a complex base band version of the signal while an external CPU is in charge to compute the $W_n$ components of the weighting vector that, in turn, depends on the steering direction. The operative bandwidth of this architecture is limited only by the velocity of the digital devices, which is expected to increase in accordance with Moore’s law. Considering a practical scenario (N/S arm) due to the size of the array (Fig. 1.4) we need to compensate for the time delays and, as a last step, for the phase delays. In fact with a length of 630 m, the loss of coherence is not acceptable (1), then the delays need to be compensated.
Fig. 1.3: Block diagram of a hardware/digital beamforming concept

Fig. 1.4: A sketch of the Northern Cross array
This compensation theoretically could be done in the digital domain using FIFOs (deep enough to handle the largest delay required by the array geometry) able to introduce (at a proper clock rate) a **COARSE** time delay, as visible in the concept block diagram shown in Fig. 1.5. The remaining part of the time delay is compensated as a pure phase delay (**FINE**) by means of the standard methods (i.e. Vector Modulator). This is known as the **Time Beamforming** method it is clear that to overcome the need to compensate the time delays (when working with large array where $\Delta t$ becomes important) it is necessary to operate with narrow bands, but this doesn’t fit with the requests of the radioastronomers who need to use even wider bands. The problem can be solved by channelling the input bandwidth as visible in Fig. 1.6. This approach is well known as **Frequency Beamforming** and offers many advantages in comparison with the time beamformer, especially when spectroscopy and pulsar observations are planned. In this last case frequency splitting is anyway requested and then implemented with a **FFT**, a **Polyphase filters** or a **PFT** (Pipelined Frequency Transform) listed here in increasing order of goodness, efficiency and cost.

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**Fig. 1.5**: Block diagram of an hardware based Time Beamformer concept. In this case we could also exploit the DDC internal registers for the phasing step.
2) **Notes on adaptive beamforming.**

The above mentioned concept is related to the process of beamforming and it doesn’t exploit the possibility to steer the nulls of the antenna beam in the direction of the RFIs (adaptive beamforming). These evolutions of standard beamformers are conceived to separate (analogically, digitally or both) a desired signal from one or more interfering signals (Spatial Filtering) by means of automatic and continuous characterization of the components of the weighting vector. This can be performed using a wide variety of different algorithms designed for many specific applications. Simple block diagrams of some concepts of adaptive beamforming are reported in Fig.2.1, 2.2 and 2.3. With these generic beamformers, the weighting vector $W$ is obtained using the signal $x(t)$ received by each element of the array (in some algorithms for special applications a reference signal is required). Some of the algorithms that have been designed will be tested in the Northern Cross SKA “test bed”, in order to choose the more suitable one for SKA applications. If the adaptive algorithms is not based on prior knowledge of the received signal (reference signal) but only based on a statistic information of the signal itself, as in the CMA (Constant Modulus Algorithm) and MV (Minimum Variance) algorithms, it is called **blind beamforming**.
Fig. 2.1: Concept of an adaptive beamforming implemented at the RF level

Fig. 2.2: A solution for a digital implementation of an adaptive digital beamforming
The MV (Minimum Variance) algorithms seems to be the most suitable to be investigated for optimum beamforming in radioastronomical applications and RFI rejection activities. In the past, (adaptive) beamforming has always been implemented in the analog domain at the RF level, but at present it is moving down to the digital domain and to the software level as in the so called “software radio architecture“ or “software array”. One of the main advantages offered by the software radio technology is flexibility. When beamforming is implemented via software, different algorithms can be tested without modifying any hardware. In this way the efforts can be concentrated on the algorithms rather than in designing new hardware [3]. A slightly more detailed block diagram of a software concept of beamformer is visible in Fig 2.4.

![Diagram](image-url)
3 Multibeaming concept.

The final concept definition of multibeaming in the context of SKA is still under discussion. We underline that, at the present point of the discussion (ISAC comments, Nov. 2002), two possible definitions of multibeaming are accepted:

- **Multibeamng** : Multiple beams within the FOV (Fig. 3.1) that can be obtained:
  - a) *On line*: Multiple beams obtained with analog or digital beamforming blocks.
  - b) *Off Line*: Multiple beams obtained with correlators and so, called synthesized beams.

![Fig. 3.1: Multiple beams within the FOV](from ISAC comments, Nov. 2002)

- **Multibeaming (Multiple field of View)**: Multiple simultaneous FOV spaced on the sky (Fig. 3.2). In the Australian lens idea of SKA this is achieved by means of multiple feeds. In the European concept (tile design), multibeaming means splitting in N parallel parts every receiver channel. These are then used to form N independent beams. Multibeaming conceived as multiple fields of view, leads to a more natural concept of **multi-user**.
We can anyway underline here some general advantages achieved by multibeam [4]:

2- Increase of the sensitivity. The improvement of the sensitivity is proportional to the square root of the number of beams. This is important when the observation needs to be performed at high sensitivity.

Increase of the total observing time (proportional to the number of beams), where the different beams can be used by several users at the same time. This way to operate is possible when the Field Of View (FOV) is large enough to allow the beams to be used at independent pointing at any time. Of course, the big amount of extra processing load requested by multibeam has to be supplied by the back end (PC cluster).

References


